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Applications of Aboodh Transform

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Abstract **- The Aboodh transform is a mathematical tool used in solving the differential equations by converting them from one form into another form. It makes it easier to solve the problem in engineering application and make differential equations simple to solve. In this paper we will discuss the application of Aboodh Transform to Simultaneous Differential Equations with boundary conditions.**

Keywords: Aboodh Transform, Differential Equations.
Sub area: Aboodh transform
Broad area: Mathematics

I. INTRODUCTION

The Aboodh transform is applied in different areas of science, engineering and technology. Ordinary linear differential equation with constant coefficient and variable coefficient can be easily solved by the Aboodh transform method without finding the generally solution and the arbitrary constant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The differential equations arising in science and engineering problems are usually solved by the methods [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]: Laplace transform, convolution method, calculus method, Elzaki Transform, Residue theorem and so on [21, 22, 23, 24, 25, 26, 27, 28, 29]. In this paper, we present Aboodh transform technique to analyze the simultaneous differential equations with boundary conditions.

II. BASIC DEFINITIONS

2.1 Aboodh Transform

If the function $h(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Aboodh transform [1-3] of $h(y)$ is given by

$$A\{h(y)\} = \bar{h}(p) = \frac{1}{p}\int_0^\infty e^{-py}\, h(y)dy.$$

The Aboodh Transform [1-3] of some of the functions are given by

- $A\{y^n\} = n!/p^{n+2}$, $where\ n = 0,1,2,..$
- $A\{e^{ay}\} = \frac{1}{p(p-a)}$,
- $A\{sinay\} = \frac{a}{p(a^2+p^2)}$,
- $A\{cosay\} = \frac{1}{a^2+p^2}$,
- $A\{sinhay\} = \frac{a}{p(p^2-a^2)}$,
- $A\{coshay\} = \frac{1}{p^2-a^2}$.
- $A\{\delta(t)\} = 1/p$

2.2 Inverse Aboodh Transform

- The Inverse Aboodh Transform [1-3] of some of the functions are given by $A^{-1}\{p^{n+2}\} = n!/y^n$

  $n = 0,1,2,3,4 \ldots$
- $A^{-1}\{\frac{1}{p(p-a)}\} = e^{ay}$
- $A^{-1}\{\frac{1}{p(a^2+p^2)}\} = \frac{1}{a}\sin ay$
- $A^{-1}\{cosay\} = \cos ay$
- $A^{-1}\{\frac{1}{p(p^2-a^2)}\} = \frac{1}{a}\sin hay$
- $A^{-1}\{\frac{1}{p^2-a^2}\} = \cos hay$

2.3 Aboodh Transform of Derivatives

The Elzaki Transform [1-3] of some of the Derivatives of h(y) are given by

- $A\{h'(y)\} = pA\{h(y)\} - h(0)/p$

  $or\ A\{h'(y)\} = p\bar{h}(p) - h(0)/p,$

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- $A\{\text{ħ}''(y)\} = p^2\bar{\text{ħ}}(p) - \frac{\text{ħ}'(0)}{p} - \text{ħ}(0), \textit{and so on.}$

III. MATERIAL AND METHOD

Simultaneous Linear Differential Equations

(A) The motion of a particle moving along a plane curve at any time t is represented by differential equations

$$\frac{dy}{dt} + 2x = sin2t\ ,$$

$$\frac{dx}{dt} - 2y = cos2t$$

Given that $x(0) = 1$, $y(0) = 0$

Solution:

The given equations can be written as

$$y' + 2x = sin2t \dots\dots\dots\dots\dots\dots(1)$$

$$x' - 2y = cos2t \dots\dots\dots\dots\dots\dots(2)$$

Taking Aboodh Transformations on both sides of (1),

$$A\{y'\} + 2A\{x\} = A\{sin2t\}$$

$$p\bar{y}(p) - \frac{y(0)}{p} + 2\bar{x}(p) = \frac{2}{p(p^2+4)}$$

$$p\bar{y}(p) + 2\bar{x}(p) = \frac{2}{p(p^2+4)} \quad \dots..(3)$$

Taking Aboodh Transformations on both sides of (2),

$$A\{x'\} - 2A\{y\} = A\{cos2t\}$$

$$p\bar{x}(p) - \frac{x(0)}{p} - 2\bar{y}(p) = \frac{1}{(p^2+4)}$$

$$p\bar{x}(p) - 2\bar{y}(p) = \frac{p^2+4+p}{p(p^2+4)} \quad \dots..(4)$$

Solving (3) and (4), we get,

$$\bar{x}(p) = \frac{p^3+p^2+4p+4}{p(p^2+4)^2}$$

$$\bar{x}(p) = \frac{p+1}{p(p^2+4)}$$

$$\bar{x}(p) = \frac{1}{(p^2+4)} + \frac{1}{2}\frac{2}{p(p^2+4)}$$

Taking Inverse Aboodh Transformations

$$x = cos2t + \frac{1}{2}\ sin2t$$

And,

$$\bar{y}(p) = -\frac{2p^2+8}{p(p^2+4)^2}$$

$$\bar{y}(p) = -\frac{2}{p(p^2+4)}$$

$$\bar{y}(p) = -sin2t$$

(B) Solve the simultaneous differential equations

$$\frac{dx}{dt} + x + \frac{dy}{dt} + y = 1$$

And

$$\frac{dy}{dt} - y - 2x = 0$$

Given that $x(0) = 0$, $y(0) = 1$

Solution:

The given equations can be written as

$$x' + x + y' + y = 1 \dots\dots\dots(1)$$

$$y' - y - 2x = 0 \ \dots\dots\dots..(2)$$

Taking Aboodh Transformations on both sides of (1),

$$A\{x'\} + A\{x\} + A\{y'\} + A\{y\} = A\{1\}$$

$$p\bar{x}(p) - \frac{x(0)}{p} + \bar{x}(p) + p\bar{y}(p) - \frac{y(0)}{p} + \bar{y}(p) = \frac{1}{p^2}$$

$$(p+1)\bar{x}(p) + (p+1)\bar{y}(p) = \frac{1}{p^2} + \frac{1}{p} \quad \dots..(3)$$

Taking Aboodh Transformations on both sides of (2),

$$A\{y'\} - A\{y\} - 2A\{x\} = 0$$

$$p\bar{y}(p) - \frac{y(0)}{p} - \bar{y}(p) - 2\bar{x}(p) = \frac{1}{p}$$

$$-2\bar{x}(p) + (p-1)\bar{y}(p) = \frac{1}{p} \quad \dots..(4)$$

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Solving (3) and (4), we get,

$$\bar{x}(p) = -\frac{1}{p^2(p+1)}$$

$$\bar{x}(p) = -\frac{1}{p^2} + \frac{1}{p^2+p}$$

Taking Inverse Aboodh Transformations

$$x = -1 + e^{-t}$$

And,

$$\bar{y}(p) = \frac{2p^2 + 2p - p^2}{p^2(p^2+p)}$$

$$\bar{y}(p) = \frac{2(p^2+p)}{p^2(p^2+p)} - \frac{p^2}{p^2(p^2+p)}$$

$$\bar{y}(p) = \frac{2}{p^2} - \frac{1}{(p^2+p)}$$

Taking Inverse Aboodh Transformations

$$y = 2 - e^{-t}$$

(C)The small oscillations of a certain system with two degrees of a freedom are given by the differential equations

$$\frac{d^2x}{dt^2} + 3x - 2y = 0$$

And

$$\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} - 3x + 5y = 0$$

Given that, $x(0) = 0$, $y(0) = 1$

$x'(0) = 3, \quad y'(0) = 2.$

Solution:

The given equations can be written as

$$x'' + 3x - 2y = 0 \ldots\ldots\ldots (1)$$

$$x'' + y'' - 3x + 5y = 0 \ \ldots\ldots (2)$$

Taking Aboodh Transformations on both sides of (1),

$$A\{x''\} + 3A\{x\} - 2A\{y\} = 0$$

$$p^2\bar{x}(p) \frac{x'(0)}{p} - x(0) + 3\bar{x}(p) - 2\bar{y}(p) = 0$$

$$(p^2+3)\bar{x}(p) - 2\bar{y}(p) = \frac{3}{p} \quad \ldots\ldots(3)$$

Taking Aboodh Transformations on both sides of (2),

$$A\{x''\} + A\{y''\} - 3A\{x\} + 5A\{y\} = 0$$

Or

$$p^2\bar{x}(p) \frac{x'(0)}{p} - x(0) + p^2\bar{y}(p) \frac{y'(0)}{p} - y(0) - 3\bar{x}(p) + 5\bar{y}(p) = 0$$

Or

$$(p^2-3)\bar{x}(p) - \bar{y}(p)(p^2+5) = \frac{5}{p} \quad \ldots\ldots(4)$$

Solving (3) and (4), we get,

$$\bar{x}(p) = \frac{3p^2+25}{(p^4+10p^2+9)p}$$

Or

$$\bar{x}(p) = \frac{11}{4}\frac{1}{p(p^2+1)} + \frac{1}{12}\frac{3}{p(p^2+9)}$$

Taking Inverse Aboodh Transformations

$$x = \frac{11}{4} sint + \frac{1}{12} sin3t$$

Also

$$\bar{y}(p) = \frac{2p^2+24}{(p^4+10p^2+9)p}$$

Or

$$\bar{y}(p) = \frac{11}{4}\frac{1}{p(p^2+1)} - \frac{1}{4}\frac{3}{p(p^2+9)}$$

Taking Inverse Aboodh Transformations

$$y = \frac{11}{4} sint - \frac{1}{4} sin3t$$

IV. CONCLUSION

The main purpose of this paper is to give a brief idea about applications of Aboodh transform in various areas and how to solve the Simultaneous Linear Differential Equations with boundary conditions. Aboodh transform is a very useful mathematical tool to make simpler complex problems.

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