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An Analytical 3-D Modeling Technique of Non-Linear Buckling Behavior of an Axially Compressed Rectangular Plate

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***Abstract* - This paper presents an analytical modeling technique for non-linear buckling behavior of axially compressed rectangular thick plate under uniformly distributed load. The aim of this study is to formulate the equation for calculation of the critical buckling load of a thick rectangular plate under uniaxial compression. Total potential energy equation of a thick plate was formulated from the three-dimensional (3-D) static elastic theory of the plate, from there on; an equation of compatibility was derived by transforming the energy equation to compatibility equation to get the relations between the rotations and deflection. The solution of compatibility equations yields the exact deflection function which was derived in terms of polynomial. The formulated potential energy was in the same way used by the method of general variation to obtain the governing differential equation whose solution gives the deflection coefficient of the plate. By minimizing the energy equation with respect to deflection coefficient after the obtained deflection and rotations equation were substituted into it, a more realistic formula for calculation of the critical buckling load was established. This expression was applied to solve the buckling problem of a thick rectangular plate that was simply supported at the first and fourth edges, clamped and freely supported in the second and third edge respectively (SCFS). Furthermore, effects of aspect ratio of the critical buckling load of a 3-D isotropic plate were investigated and discussed. The numerical analysis obtained showed that, as the aspect ratio of the plate increases, the value of critical buckling load decreases while as critical buckling load increases as the length to breadth ratio increases. This implies that an increase in plate width increases the chance of failure in a plate structure. It is concluded that as the in-plane load which will cause the plate to fail by compression increases from zero to critical buckling load, the buckling of the plate exceeds specified elastic limit thereby causing failure in the plate structure.**

Keywords: uniaxial buckling, CSFS rectangular plate, compressive load, variational method, stability analysis of thick plate, three-dimensional (3-D) plate theory.

I. INTRODUCTION

Plates are three dimensional structural elements widely used in architectural structures and various engineering applications such as floor slabs, bridge decks, rigid pavements of highways and airport runways, ship decks, aircraft and spacecraft panels and retaining walls [1, 2]. In the design of engineering structures, stability is a factor of great importance that must be considered and it is especially true for structures with one or two dimensions that are small in relation to the other dimensions, such as plates. A plate is a solid that consists of two parallel plane surfaces separated by a small dimension called thickness [3, 4]; compared with the thickness, the planer surface dimensions are large. Based on shapes, plates can be rectangular, square, triangular circular, elliptical, circular with hole, or square with hole. They can also be isotropic, anisotropic, orthotropic, homogeneous, and heterogeneous, regarding the materials of construction. Based on weight, plates can be thin or thick [5, 6, 7]. At the edges, plates have varying boundary conditions; for a rectangular plate, they are clamped support, simply supported and free edged.

Plates can exhibit flexural, dynamic, as well as buckling behaviors and the behaviors of plates depend on the type and nature of load application [8, 9]. Instability of structures is commonly characterized with buckling [10]. A structure is said to buckle when it encounters large deformation and loses its ability to withstand the load at a critical load value. With the increased use of thick plates in engineering projects, stability analysis of plate structures is required.

The stability analysis of plates has attracted great research interest with many varying methods being developed and applied. Considering the insufficient ability of plates towards withstanding compressive forces in structures carrying in-plane compressive loads, the necessity of stability analysis is undeniable. Buckling of plates can be analyzed using numeric, equilibrium or energy methods [11, 12]. Meanwhile, considering different theories of plate analysis, unlike 2-D, more attention has not been channeled to a typical 3-D elasticity theory of plate because of their rigorous mode of

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analysis as they considered all the stress element in the analysis.

For a rectangular thin plate with two simply supported edge, one clamped and free support edge (SSCF) and all edges simply supported plate (SSSS) subjected uniaxial uniform compressive loads, finite element method was used by Persson *et al.* [13] to solve the problem of elastic stability. Zureick [14] studied the buckling load analysis in a simply supported thick isotropic square plate thick, which was subjected to biaxial and uniaxial in-plane forces, by applying shear exponential deformation theory. Both [13] and 14] used finite element which performs a detailed numerical study, however, their result is approximate and cannot be reliably as they will not acutely predict the buckling load of the plate. They did not consider a thick plate with SCFS boundary condition and polynomial shape function was not regarded. Considering the ease in mathematical manipulation, applying the polynomial expression as the deflection function simplifies buckling analysis.

Onwuka *et al.* [15] used the Galerkin's method to analyze buckling in all-edge simply supported thin rectangular isotropic plates. Polynomial series was used to obtain the plate equation of deflection, neither was a thick plate with SCFS boundary condition considered. Although the energy approach was used, the authors did not perform the general variation to the energy equation to determine the exact deflection function rather the assumed a shape function which is unreliable for proper estimation of buckling load of the plate.

Ibearugbulem *et al.* [16] carried out a study on the buckling analysis of axially compressed rectangular plate by using Taylor-Mclaurin's series in its theoretical formulation, thereafter; Ritz approach was applied to determine the numerical field of unknown function of thin rectangular flat SSSS plate. They limited their study on thin plate; the result cannot solve for thick and moderately thick plate. Tamijani *et al.* [17], used the same approach for the buckling and vibration analysis of thick plate. Both authors [16, 17] assumed a shape function which can only give an approximate solution, hence, cannot be reliable in the plate's analysis as they might under-estimate the buckling load at an improved thickness of the plate.

Onyeka *et al.* [18] studied an exact bending feature of an elastic rectangular three-dimensional thick plate with all edges simply supported (SSSS) and carrying uniformly distributed transverse load using the using the direct variation energy technique. By developing a trigonometric model as the plate shape function they determine the displacement and stress expression for the bending analysis. They neither got the buckling load that may occur due to compression of the plate nor solve for the present boundary condition. More so, they did not consider the polynomial shape function which can be easily applied to any support case.

Onyeka *et al.* [19 and 20] used the variation energy approach in a 3D buckling analysis of a one edge clamped and other edges simply supported (CSSS), and all-edge-clamped (CCCC) rectangular isotropic plate under compressive uniaxial load. They developed a new trigonometric shear deformation plate theory that is capable of analyzing any category of the plate. They did not consider the polynomial displacement function as the plates shape function for the analysis. Onyeka *et al.* [21] used the same approach to study the stability of thick that was simply supported at all the four edges (SSSS). Both authors [19, 20 and 21] did not study the buckling analysis of 3-D rectangular plates that can handle any type of plate (thin, thick and moderately thick) under the present boundary condition.

There exists an aspect of distinctiveness of the present study over the previous works put together. This includes; the method of analysis, type of shape functions, and plates support boundary conditions. Unlike the previous works that assumed the displacement function, the present work derives the expression for the solution of the problem from the first principle of elasticity to get an exact polynomial displacement function. The research gap as reviewed in literatures was filled with this study by developing a non-linear 3-D modeling technique for the buckling behavior of a thick rectangular plate under uniaxial compression using an analytical approach. This study will evaluate the effect of aspect ratio of the critical buckling load of a thick rectangular plate that was simply supported at the first and fourth edges, clamped and freely supported at the second and the third edge (SCFS) using a 3-D elastic plate theory. This is to eliminate structural failure in the plate by ensuring that the buckling of the plate does not exceed specified elastic limit.

II. METHODOLOGY

2.1 Potential Energy Equation Formulation

The energy equation for an axially loaded rectangular thick plate is formulated by considering a thick plate assumption, with the x-z section and y-z section, which are initially normal to the x-y plane before bending off the normal to the x-y plane after bending of the plate as shown in the section of plate presented in the figure 1.

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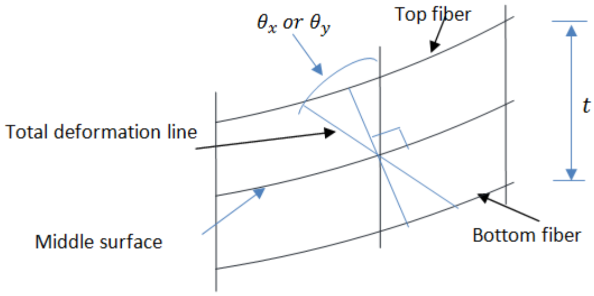

Figure 1: Rotation of x-z (or y-z) section after bending

The non-dimensional total potential energy [∏]expression for an elastic three-dimensional plate theory of R and Q coordinates at the span-thickness aspect ratio (a/t) is in line with [20] and presented as:

$$\prod = D^{*}\frac{(1-\mu)ab}{2a^{2}(1-2\mu)}\int_{0}^{1}\int_{0}^{1}\Bigg[(1-\mu)\left(\frac{\partial\theta_{sx}}{\partial R}\right)^{2}+\frac{1}{\beta}\frac{\partial\theta_{sx}}{\partial R}.\frac{\partial\theta_{sy}}{\partial Q}+\frac{(1-\mu)}{\beta^{2}}\left(\frac{\partial\theta_{sy}}{\partial Q}\right)^{2}+\frac{(1-2\mu)}{2\beta^{2}}\left(\frac{\partial\theta_{sx}}{\partial Q}\right)^{2}+\frac{(1-2\mu)}{2}\left(\frac{\partial\theta_{sy}}{\partial R}\right)^{2}$$
$$+\frac{6(1-2\mu)}{t^{2}}\left(a^{2}\theta_{sx}{}^{2}+a^{2}\theta_{sy}{}^{2}+\left(\frac{\partial w}{\partial R}\right)^{2}+\frac{1}{\beta^{2}}\left(\frac{\partial w}{\partial Q}\right)^{2}+2a.\theta_{sx}\frac{\partial w}{\partial R}+\frac{2a.\theta_{sy}}{\beta}\frac{\partial w}{\partial Q}\right)+\frac{(1-\mu)a^{2}}{t^{4}}\left(\frac{\partial w}{\partial S}\right)^{2}$$
$$-\frac{N_x}{D^{*}}.\left(\frac{\partial w}{\partial R}\right)^{2}\Bigg]\partial R\partial Q \qquad (1)$$

Given that D^{*} is the Rigidity for 3-D thick plate, let

$$D^{*} = D\,\frac{(1-\mu)}{(1-2\mu)}$$

Where D is the Rigidity of the CPT or incomplete 3-D thick plate, let

N_x, μ, w, θ_{Sx}, and θ_{Sy} are the uniform applied uniaxial compression load of the plate, the poison ratio, deflection , shear deformation rotation along x axis and shear deformation rotation along y axis respectively.

2.2 Compatibility Equation

The true compatibility equations in x-z planeand y-z plane according to [21] is obtained by minimizing the energy equation with respect to rotation in x-z plane and rotation in y-z plane and equate its integrands to zero to get:

$$(1-\mu)\frac{\partial^{2}\theta_{sx}}{\partial R^{2}}+\frac{1}{2\beta}.\frac{\partial^{2}\theta_{sy}}{\partial R\partial Q}+\frac{(1-2\mu)}{2\beta^{2}}\frac{\partial^{2}\theta_{sx}}{\partial Q^{2}}+\frac{6(1-2\mu)}{t^{2}}\left(a^{2}\theta_{sx}+a.\frac{\partial w}{\partial R}\right)=0 \qquad (2)$$

$$\frac{1}{2\beta}.\frac{\partial^{2}\theta_{sx}}{\partial R\partial Q}+\frac{(1-\mu)}{\beta^{2}}\frac{\partial^{2}\theta_{sy}}{\partial Q^{2}}+\frac{(1-2\mu)}{2}\frac{\partial^{2}\theta_{sy}}{\partial R^{2}}+\frac{6(1-2\mu)}{t^{2}}\left(a^{2}\theta_{sy}+\frac{a.}{\beta}\frac{\partial w}{\partial Q}\right)=0 \qquad (3)$$

Using the law of addition, Equations 2 and 3 will be simplified, then factorizing the outcome gives:

$$\frac{\partial w}{\partial R}\left[(1-\mu)\frac{\partial^{2}}{\partial R^{2}}+\frac{1}{\beta^{2}}.\frac{\partial^{2}}{\partial Q^{2}}(1-\mu)+\frac{6(1-2\mu)a^{2}}{t^{2}}.\left(1+\frac{1}{c}\right)\right]=0 \qquad (4)$$

$$\frac{1}{\beta}.\frac{\partial w}{\partial Q}\left[\frac{\partial^{2}}{\partial R^{2}}(1-\mu)+\frac{(1-\mu)}{\beta^{2}}\frac{\partial^{2}}{\partial Q^{2}}+\frac{6(1-2\mu)a^{2}}{t^{2}}.\left(1+\frac{1}{c}\right)\right]=0 \qquad (5)$$

After simplification using law of addition, one of the possible of Equation becomes:

$$\frac{6(1-2\mu)(1+c)}{t^{2}}=-\frac{c(1-\mu)}{a^{2}}\left(\frac{\partial^{2}}{\partial R^{2}}+\frac{1}{\beta^{2}}\frac{\partial^{2}}{\partial Q^{2}}\right) \qquad (6)$$

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2.3 General Governing Equation

The minimization of energy equation with respect to deflection gives the general governing equation as presented in [22]:

$$\frac{D^*}{2a^2}\int_0^1\int_0^1\left[\frac{6(1-2\mu)(1+c)}{t^2}\left(\frac{\partial^2 w}{\partial R^2}+\frac{1}{\beta^2}\cdot\frac{\partial^2 w}{\partial Q^2}\right)+\frac{(1-\mu)a^2}{t^4}\frac{\partial^2 w}{\partial S^2}-\frac{N_x}{D^*}\cdot\frac{\partial^2 w}{\partial R^2}\right]dR\,dQ=0 \qquad (7)$$

Substituting Equation 6 into Equation 7 and simplifying the outcome gives two governing differential equations of a 3-D rectangular plate subject to pure buckling as presented in Equation 8 and 9:

$$\frac{\partial^4 w_1}{\partial R^4}+\frac{2}{\beta^2}\cdot\frac{\partial^4 w_1}{\partial R^2\partial Q^2}+\frac{1}{\beta^4}\cdot\frac{\partial^4 w_1}{\partial Q^4}-\frac{N_{x1}a^4}{gD^*}\cdot\frac{\partial^2 w_1}{\partial R^2}=0 \qquad (8)$$

$$\frac{(1-\mu)a^4}{t^4}\cdot\frac{\partial^2 w_S}{\partial S^2}-\frac{N_{xs}a^4}{D^*}\cdot\frac{\partial^2 w_S}{\partial R^2}=0 \qquad (9)$$

Thus, the polynomial expression for deflection derived from Equation (8) according to Onyeka *et al.* [23]is presented in Equation (10) as:

$$w=\Delta_0(a_0+\ a_1R+a_2R^2+a_3R^3+\ a_4R^4)\times(b_0+\ \ b_1Q+\ b_2Q^2+b_3Q^3+b_4Q^4) \qquad (10)$$

Equation (10) can be re-written in line with the work of Onyeka *et al.* [23] as:

$$w=A_1h \qquad (11)$$

Where:

$$A_1=\Delta_0\begin{bmatrix}a_0\\a_1\\a_2\\a_3\\a_4\end{bmatrix}.\begin{bmatrix}b_0\\b_1\\b_2\\b_3\\b_4\end{bmatrix} \qquad (12)$$

$$h=[1\ \ RR^2R^3R^4].[1\ \ QQ^2Q^3Q^4] \qquad (13)$$

$$\theta_{sx}=\frac{A_2}{a}\cdot\frac{\partial h}{\partial R} \qquad (14)$$

$$\theta_{sy}=\frac{A_3}{a\beta}\cdot\frac{\partial h}{\partial Q} \qquad (15)$$

Given that:$h$is the shape function of the plate, A_1 is the coefficient of deflection$A_2$and$A_3$are the coefficients of shear deformation in x axis and y axis respectively.

2.4 Direct Governing Equation

By substituting Equations (11), (14) and (15) into Equation (1), the Energy equation becomes:

$$\Pi=\frac{D^*ab}{2a^4}\Bigg[(1-\mu){A_2}^2\int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R^2}\right)^2 dRdQ+\frac{1}{\beta^2}\left[A_2.A_3+\frac{(1-2\mu){A_2}^2}{2}+\frac{(1-2\mu){A_3}^2}{2}\right]\int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial R\partial Q}\right)^2+\frac{(1-\mu){A_3}^2}{\beta^4}\int_0^1\int_0^1\left(\frac{\partial^2 h}{\partial Q^2}\right)^2 dRdQ$$
$$+6(1-2\mu)\left(\frac{a}{t}\right)^2\left([{A_2}^2+{A_1}^2+2A_1A_2].\int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2 dRdQ+\frac{1}{\beta^2}.[{A_3}^2+{A_1}^2+2A_1A_3].\int_0^1\int_0^1\left(\frac{\partial h}{\partial Q}\right)^2 dRdQ\right)$$
$$-\frac{N_xa^2{A_1}^2}{D^*}.\int_0^1\int_0^1\left(\frac{\partial h}{\partial R}\right)^2 dRdQ\Bigg] \qquad (16)$$

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Differentiating Equation (17) with respect to shear deformation coefficient (A_2 and A_3), and solve simultaneously gives:

$$A_2 = \left(\frac{k_{12}k_{23} - k_{13}k_{22}}{k_{12}k_{12} - k_{11}k_{22}}\right).A_1 \qquad (17)$$

$$A_3 = \left(\frac{k_{12}k_{13} - k_{11}k_{23}}{k_{12}k_{12} - k_{11}k_{22}}\right).A_1 \qquad (18)$$

Let:

$$k_{11} = (1-\mu)k_{RR} + \frac{1}{2\beta^2}(1-2\mu)k_{RQ} + 6(1-2\mu)\left(\frac{a}{t}\right)^2 k_R \qquad (19)$$

$$k_{21} = k_{12} = \frac{1}{2\beta^2}k_{RQ};\ k_{13} = -6(1-2\mu)\left(\frac{a}{t}\right)^2 k_R;\ k_{32} = k_{23} = -\frac{6}{\beta^2}(1-2\mu)\left(\frac{a}{t}\right)^2 k_Q \qquad (20)$$

$$k_{22} = \frac{(1-\mu)}{\beta^4}k_{QQ} + \frac{1}{2\beta^2}(1-2\mu)k_{RQ} + \frac{6}{\beta^2}(1-2\mu)\left(\frac{a}{t}\right)^2 k_Q \qquad (21)$$

Differentiating Equation (16) with respect to deflection coefficient (A_1) and simplifying the outcome, an expression for the critical buckling load(N_{xcr}) is established as:

$$\frac{N_x a^2}{D^*} = 6(1-2\mu)\left(\frac{a}{t}\right)^2\left(\left[1 + \left(\frac{k_{12}k_{23} - k_{13}k_{22}}{k_{12}k_{12} - k_{11}k_{22}}\right)\right] + \frac{1}{\beta^2}.\left[1 + \left(\frac{k_{12}k_{13} - k_{11}k_{23}}{k_{12}k_{12} - k_{11}k_{22}}\right)\right].\frac{k_Q}{k_R}\right) \qquad (22)$$

Similarly:

$$N_{xcr} = \frac{(1+\mu)Et^3}{2a^2}\left(\frac{a}{t}\right)^2\left(\left[1 + \left(\frac{k_{12}k_{23} - k_{13}k_{22}}{k_{12}k_{12} - k_{11}k_{22}}\right)\right] + \frac{1}{\beta^2}.\left[1 + \left(\frac{k_{12}k_{13} - k_{11}k_{23}}{k_{12}k_{12} - k_{11}k_{22}}\right)\right].\frac{k_Q}{k_R}\right) \qquad (23)$$

Where;

E is the modulus of elasticity and β represents the ratio of length and breadth of the plate.

2.5 Numerical Analysis

A problem of a rectangular thick plate that is simply supported at the first and fourth edges, clamped and freely supported in the second and third edge respectively (SCFS) under uniaxial compressive load is presented. The trigonometric function (see Equation (10)) was used different aspect ratios to determine the value of the critical buckling load of the plate presented in Figure 2.

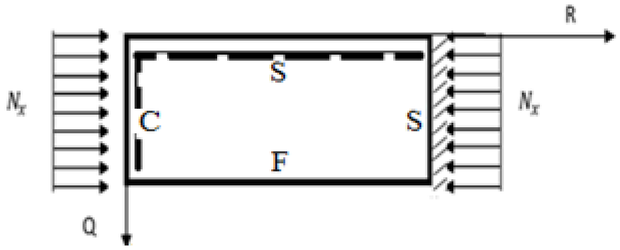

Figure 2: SCFS Rectangular Plate subjected to uniaxial compressive load

The boundary conditions of the plate in figure 2 are as follows:

$$\text{At } R = Q = 0; \text{deflection } (w) = 0 \qquad (24)$$

$$\text{At } R = 0, \text{ bending moment } \left(\frac{d^2w}{dR^2}\right) = 0;\ Q = 0, \text{slope } \left(\frac{dw}{dQ}\right) = 0 \qquad (25)$$

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At $R = 1$, deflection $(w) = 0$; $Q = 1$, bending moment $\left(\frac{d^2w}{dQ^2}\right) = 0$ (26)

At $R = 1$, bending moment $\left(\frac{d^2w}{dR^2}\right) = 0$; Q = 1, shear force $\left(\frac{d^3w}{dQ^3}\right) = 0$ (27)

At Q = 1, slope $\left(\frac{dw}{dQ}\right) = \frac{2}{3b_5}$ (28)

Substituting Equation (24) to (28) into the derivatives of w and solving gave the characteristic equations with the following constants:

$$a_0 = 0;\ a_1 = 0;\ a_2 = 1.5a_4;\ a_3 = -2.5a_4\ and \quad (29)$$
$$b_0 = 0;\ b_1 = -\frac{7}{3}b_5;\ b_2 = 0;\ b_3 = -\frac{10b_5}{3};\ b_4 = -\frac{10b_5}{3} \quad 30)$$

Substituting the constants of Equation (29) and (30) into Equation (10) gives;

$$w = (1.5a_4R^2 - 2.5a_4R^3 + a_4R^4) \times \left(\frac{7b_5Q}{3} - \frac{10b_5}{3}Q^3 + \frac{10b_5}{3}Q^4 - b_5Q^5\right) \quad (31)$$

Simplifying Equation (31) which satisfying the boundary conditions of Equation (24 to 28) gives:

$$w = a_4(1.5R^2 - 2.5R^3 + R^4) \times b_5\left(\frac{7Q}{3} - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \quad (32)$$

Let the amplitude,

$$A_1 = a_4 \times b_5 \quad (33)$$

And;

$$h = (1.5R^2 - 2.5R^3 + R^4) \times \left(\frac{7Q}{3} - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right) \quad (34)$$

Thus, the polynomial deflection functions after satisfying the boundary conditions is:

$$w = (1.5R^2 - 2.5R^3 + R^4) \times \left(\frac{7Q}{3} - \frac{10}{3}Q^3 + \frac{10}{3}Q^4 - Q^5\right).A_1 \quad (35)$$

As such, numerical values of the stiffness for a SCFS plate were obtained using Equation (24) to (28) by applying the polynomial function as obtained in Equation (34) and their results are presented in Table 1.

Table 1: The polynomial and trigonometric stiffness coefficients of deflection function of the SCFS plate

Displacement Shape Function	k_{RR}	k_{RQ}	k_{QQ}	k_R	k_Q
Polynomial	0.32848	0.09190	0.12867	0.03324	0.00931

III. RESULTS AND DISCUSSIONS

In this section, a numerical value of the buckling load expression obtained in Equation (23) is presented. The non-dimensional value of the critical buckling load for an isotropic rectangular plate that was simply supported at the first and fourth edges, clamped and freely supported in the second and the third edge (SCFS) under uniaxial compressive load at varying aspect ratio is presented in Table 2. This result was obtained by expressing the shape function of the plate in the form of polynomial to obtain the critical buckling load of the plate. A numerical and graphical representation was presented in Figure 3 to 7 to show the behavior of the element a thick plate's stability at varying thickness and aspect ratio.

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The values obtained in Table 2, shows that as the values of critical buckling load increase, the span- thickness ratio increases. This reveals that as the in-plane loads on the plate increase and approaches the critical buckling, the failure in a plate structure is a bound to occur; this means that a decrease in the thickness of the plate increases the chance of failure in a plate structure. Hence, failure tendency in the plate structure can be mitigated by increasing its thickness.

It is also observed in the tables that as the aspect ratio of the plate increases, the value of critical buckling load decreases while as critical buckling load increases as the length to breadth ratio increases. This implies that an increase in plate width increases the chance of failure in a plate structure. It can be deduced that as the in-plane load which will cause the plate to fail by compression increases from zero to critical buckling load, the buckling of the plate exceeds specified elastic limit thereby causing failure in the plate structure. This meant that, the load that causes the plate to deform also causes the plate material to buckle simultaneously.

Thus, the result of the present model which analyzed all the stress elements in the plate is considered safer to use to achieve an exact three-dimensional plate analysis using polynomial displacement functions hence, provides accurate or reliable solution in the analysis of a rectangular plate under SCFS boundary condition.

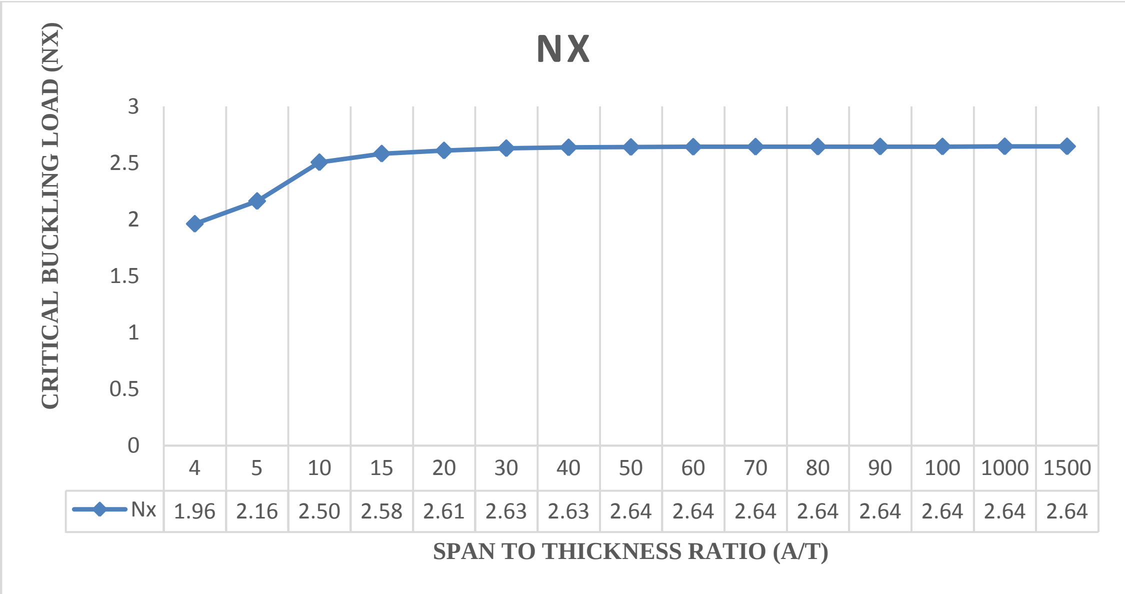

Figure 3: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a square rectangular plate

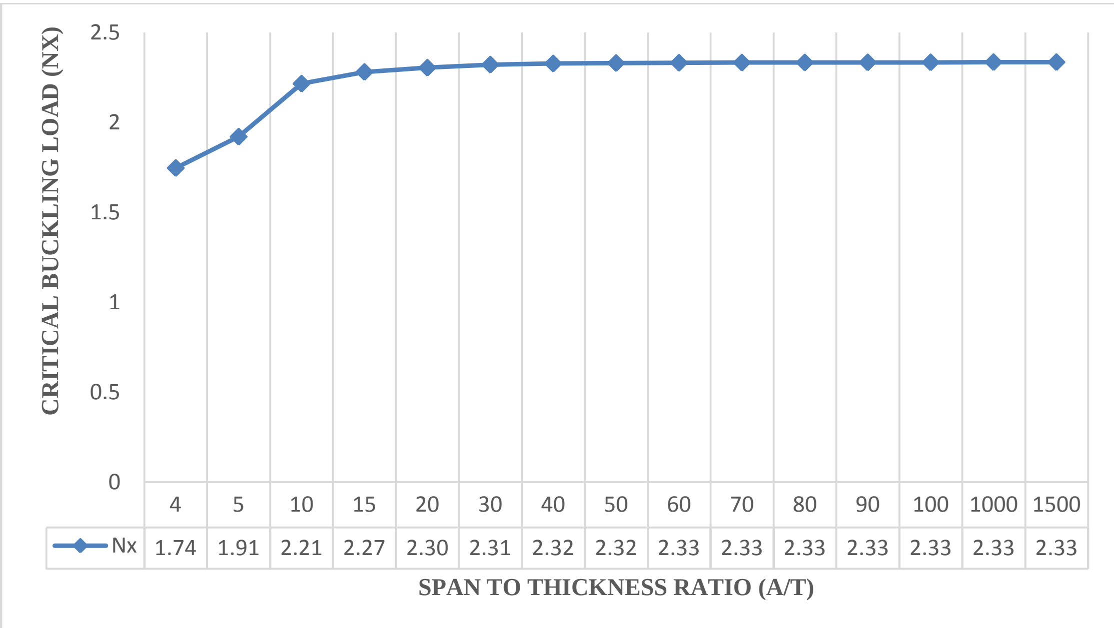

Figure 4: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 1.5

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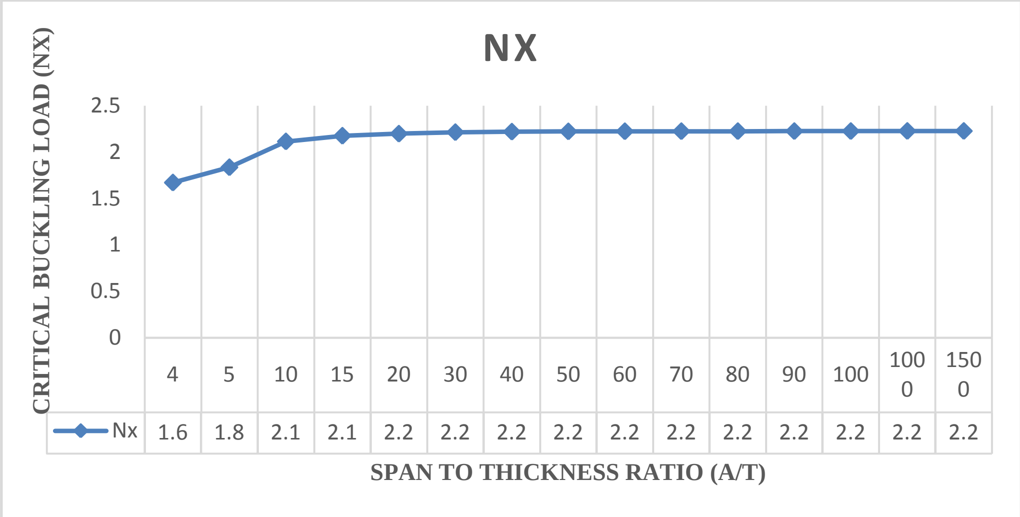

Figure 5: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 2.0

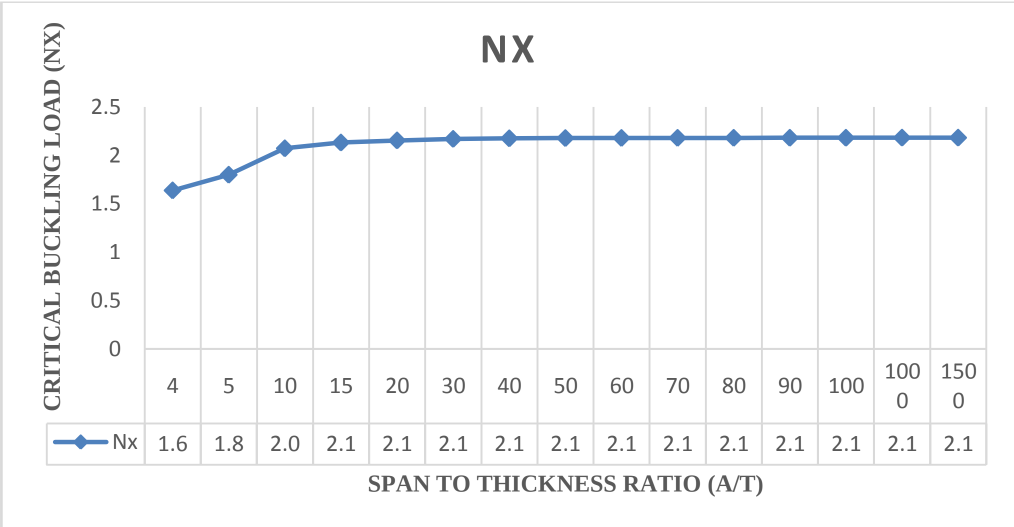

Figure 6: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 2.5

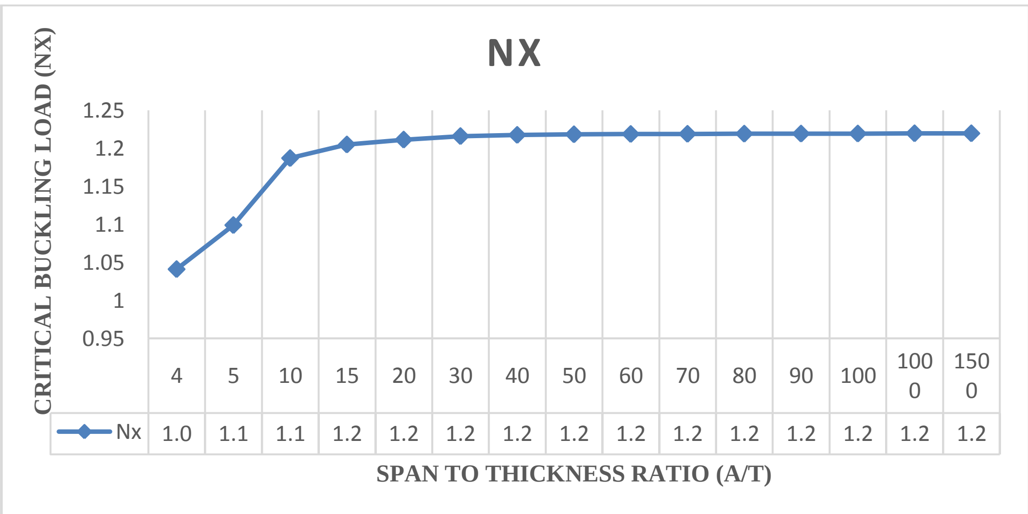

Figure 7: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 3.0

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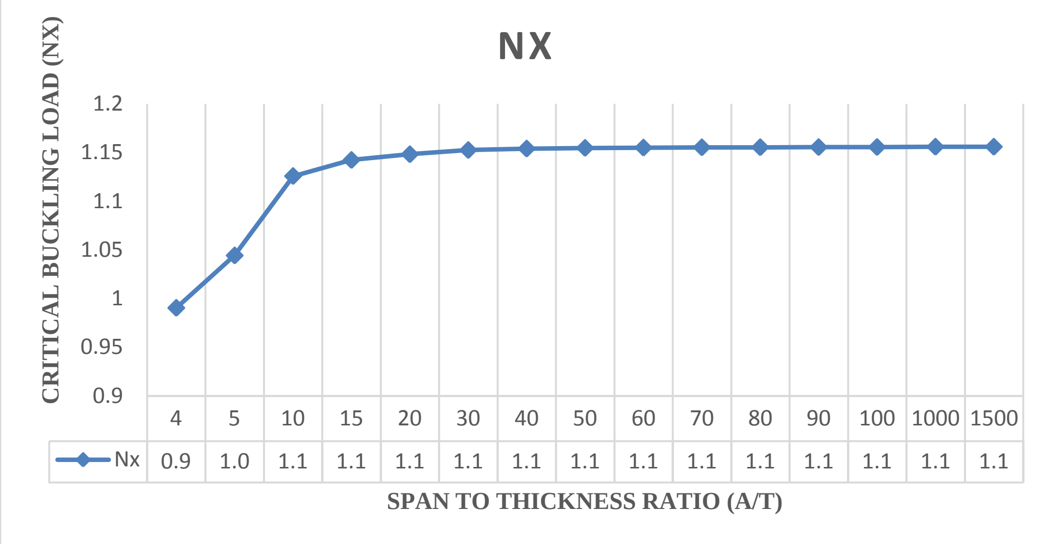

Figure 8: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 3.5

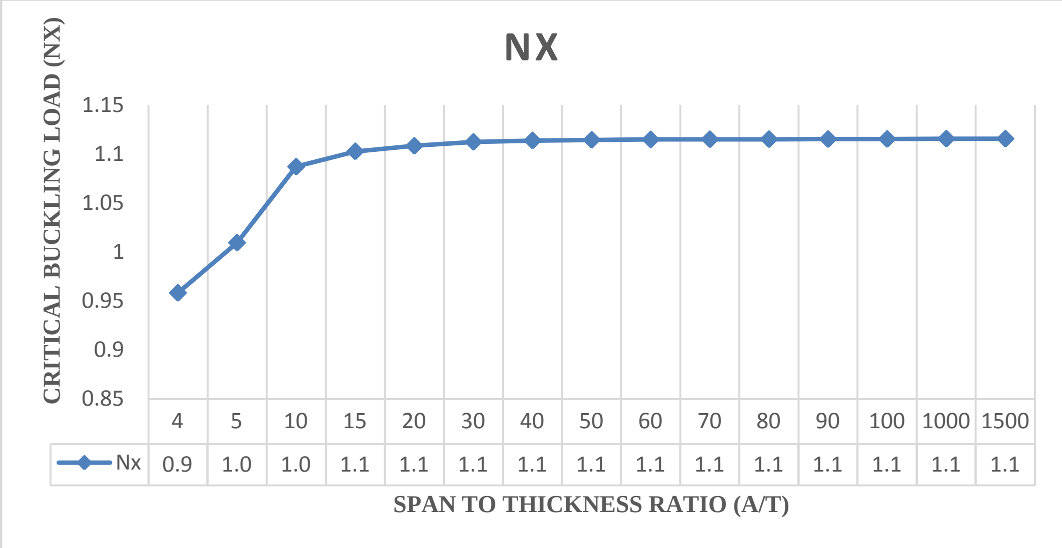

Figure 9: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 4.0

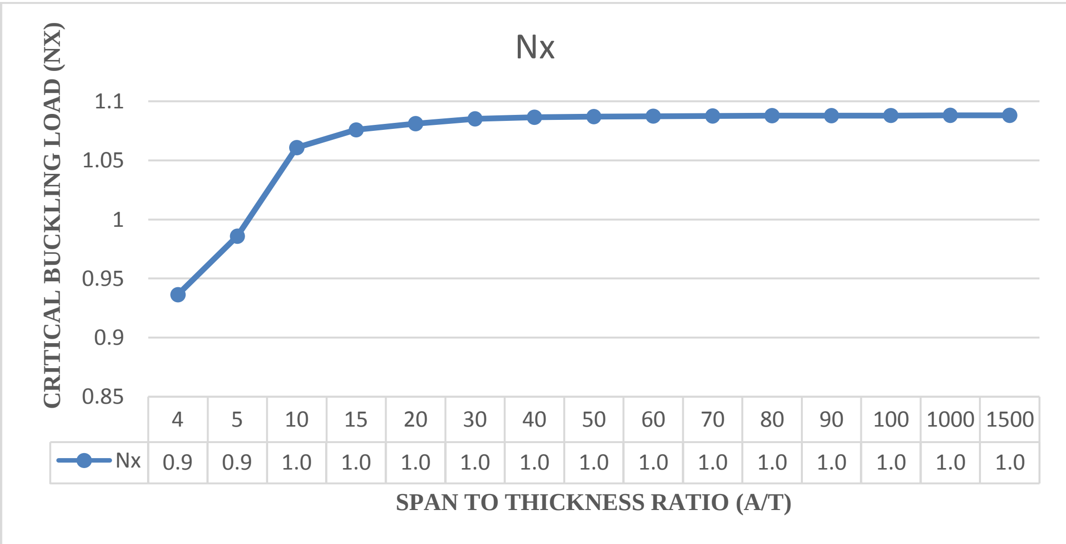

Figure 10: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 4.5

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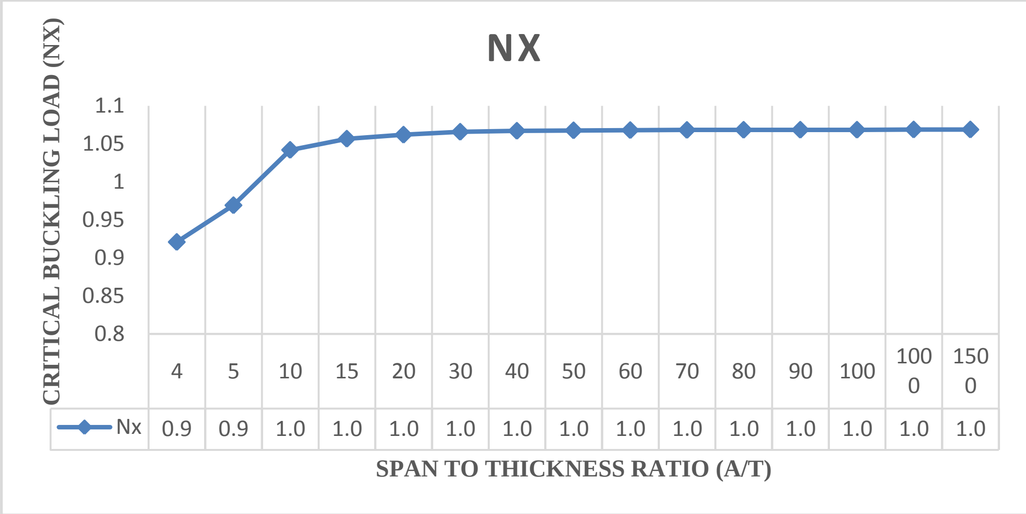

Figure 11: Critical buckling load (N_x) versus span to thickness ratio (a/t) of a rectangular plate with aspect ratio of 5.0

IV. CONCLUSION AD RECOMMENDATION

From the result of the analysis of this study, it can be concluded that an increase in plate width increases the chance of failure in a plate structure while the increase in the plate thickness ensures safety in the plate structure subjected to uniaxial compression. Furthermore, it is shown that as the in-plane load which will cause the plate to fail by compression increases from zero to critical buckling load, the buckling of the plate exceeds specified elastic limit thereby causing failure in the plate structure. More, so, the polynomial displacement function developed to give a close form solution, thereby considered more accurate and safe for complete exact three-dimensional thick plate analysis than the polynomial. Its use in the analysis of thick plates will yield almost an exact result. Thus, proof that the 3-D plate theory provides a reliable solution in the stability analysis of plates and, can be recommended for analysis of any type of rectangular plate under support condition and load configuration.

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