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On the Exact Soliton Solution for the Generalized Klein-Gordon Equation

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***Abstract* - In this paper, we established a traveling wave solution by using ansatz method say, Sine-function method for the generalized Klein-Gordon equation.**

Keywords: Sine-function method, generalized Klein–Gordon equation, solution solutions.

I. INTRODUCTION

Investigation of exact solitary wave solutions of the nonlinear evolution equations (NLEEs) has a vital role in the current study of nonlinear complex physical phenomena. In past few decades, a number of very powerful and direct methods have been proposed and developed to find the explicit solutions of NLEEs, such as the tanh-method [1], the homotopy perturbation method [2], the homogeneous balance method [3], the sine–cosine method [4], the improved F-expansion method [5], exp-function method [6], (G'/G)-expansion method [7], the simplest equation method [8], Lie symmetry method [9–14], Hirota Bilinear method [15], the generalized hyperbolic-function [16,17], the separation of variables method [18.19], First Integral Method [20,21] and so on.

The Klein–Gordon equation plays an important role in mathematical physics. The equation has attracted much attention in studying solitons [22] in condensed matter physics, in investigating the interaction of solitons in a collision less plasma, and the recurrence of initial states. The generalized nonlinear Klein–Gordon equation is given by

$u_{tt} - p^2 u_{xx} + au - bu^{\gamma} = 0$, (with nonlinear term of order $\gamma \in \Re,\ \gamma \neq \pm 1$).

When γ takes the values 2 and 3, generalized nonlinear Klein–Gordon equation becomes the nonlinear Klein–Gordon equation with quadratic and cubic nonlinear term, respectively. Various methods have been used for solving generalized nonlinear Klein–Gordon equation with $\gamma = 2, 3$. In [23], Adomian's Decomposition scheme is used for solving nonlinear Klein–Gordon equation with $\gamma = 3$. Liu et al. [24], used Jacobi elliptic function expansion method to construct travelling wave solutions for nonlinear Klein–Gordon equation with $\gamma = 2, 3$. Also, in [25], the author used an auxiliary ordinary differential equation to generate new exact travelling wave solutions for nonlinear Klein–Gordon equation with $\gamma = 2, 3$. Moreover Zhang [26], used the extended Jacobi elliptic function expansion method to solve nonlinear Klein–Gordon equation with $\gamma = 2$ and some new exact solutions are obtained. In this paper, we aim to find exact solitary wave solutions for the generalized Klein–Gordon equation.

II. THE SINE FUNCTION METHOD

Consider the nonlinear partial differential equation of the form

$$F(u, u_t, u_x, u_{xx}, u_{xxt}, \dots) = 0 \qquad (1)$$

Where $u(x,t)$ is the solution of nonlinear partial differential equation (1). We use the transformations,

$$u(x,t) = f(\xi), \quad \xi = x - ct \qquad (2)$$

This enables us to use the following changes:

$$\frac{\partial}{\partial t}(\cdot) = -c\frac{d}{d\,\xi}(\cdot),\ \frac{\partial}{\partial x}(\cdot) = \frac{d}{d\,\xi}(\cdot),\ \frac{\partial^2}{\partial x^2}(\cdot) = \frac{d^2}{d\,\xi^2}(\cdot), \dots \qquad (3)$$

Eq. (3) changes Eq. (1) in the form

$$G(f, f', f'', f''', \dots) = 0 \qquad (4)$$

The solution of Eq. (4) can be expressed in the form:

$$f(\xi) = \lambda\, sin^{\alpha}(\mu\,\xi), \qquad |\xi| \le \frac{\pi}{\mu} \qquad (5)$$

Where λ, α and μ are unknown parameters which are to be determined. Thus we have:

$$f' = \frac{df(\xi)}{d\xi} = \lambda\,\alpha\,\mu\, sin^{\alpha-1}(\mu\,\xi)\cos(\mu\,\xi) \qquad (6)$$

$$f'' = \frac{d^2 f(\xi)}{d\xi^2} = -\lambda\,\mu^2\alpha\, sin^{\alpha}(\mu\xi) + \lambda\,\mu^2\alpha(\alpha-1)sin^{\alpha-2}(\mu\xi) - \lambda\,\mu^2\alpha(\alpha-1)sin^{\alpha}(\mu\xi) \qquad (7)$$

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Substituting Eq. (5) in Eq. (4) gives a trigonometric equation of $sin^{\alpha}(\mu\,\xi)$ terms. To determine the parameters first balancing the exponents of each pair of sine to find α. Then collecting all terms with the same power in $sin^{\alpha}(\mu\,\xi)$ and put to zero their coefficients to get a system of algebraic equations among the unknowns λ, α and μ. Now the problem is reduced to a system of algebraic equations that can be solved to obtain the unknown parameters λ, α and μ. Hence, the solution considered in Eq. (5) is obtained. The above analysis yields the following theorem:

Theorem: The exact analytical solution of the nonlinear partial differential equations Eq. (1) can be determined in the form Eq. (5) where all constants found from the algebraic equations after its solutions.

III. APPLICATION

In order to illustrate the effectiveness of the proposed method we will consider the generalized nonlinear Klein–Gordon equation in the following form:

$u_{tt} - p^2 u_{xx} + au - bu^{\gamma} = 0$, (with nonlinear term of order $\gamma \in \Re,\ \gamma \neq \pm 1$). (8)

Using the transformation, $u(x,t) = f(\xi),\quad \xi = x - ct$, Eq. (8) reduces to:

$$(c^2 - p^2)f'' + a\,f - bf^{\gamma} = \qquad (9)$$

Substituting Eq. (5) and (7) into (9) gives:

$$-(c^2 - p^2)\lambda\alpha\mu^2\, sin^{\alpha}(\mu\,\xi) + (c^2 - p^2)\lambda\mu^2\alpha(\alpha-1)\, sin^{\alpha-2}(\mu\,\xi) - (c^2 - p^2)\lambda\mu^2\alpha(\alpha-1)\, sin^{\alpha}(\mu\,\xi)$$

$$+a\lambda\, sin^{\alpha}(\mu\,\xi) - b\lambda^{\gamma}\, sin^{\gamma\alpha}(\mu\,\xi) = 0 \qquad (10)$$

Eq. (10) is satisfied only if the following system of algebraic equations holds:

$$\gamma\alpha = \alpha - 2,$$

$$-(c^2 - p^2)\lambda\alpha\mu^2 - (c^2 - p^2)\lambda\mu^2\alpha(\alpha-1) + a\lambda = 0,$$

$$-b\lambda^{\gamma} + (c^2 - p^2)\lambda\mu^2\alpha(\alpha-1) = 0. \qquad (11)$$

Solving the system of equations (11), we obtain:

$$\alpha = \frac{2}{1-\gamma},\quad \mu = \pm\sqrt{\frac{a(1-\gamma)}{2(c^2-p^2)}},\quad and\ \ \lambda = \left[\frac{a(\alpha-1)}{b}\right]^{\frac{1}{\gamma-1}} \qquad (12)$$

Substituting Eq. (13) into Eq. (5) we obtain the exact soliton solution of the generalized nonlinear Klein–Gordon equation,

$$u(x,t) = \left[\frac{a(\alpha-1)}{b}\right]^{\frac{1}{\gamma-1}} sin^{\frac{2}{1-\gamma}}\left(\pm\sqrt{\frac{a(1-\gamma)}{2(c^2-p^2)}}\,(x-ct)\right) \qquad (13)$$

This gives the desired exact soliton solution of the generalized nonlinear Klein–Gordon equation.

IV. CONCLUSION

In this paper, the ansatz method, sine-function method has been successfully applied to find the solution for generalized nonlinear Klein–Gordon equation. The sine-function method is used to find new exact solution. Thus, it is possible that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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