The Theoretical and Empirical Equivalence of Cronbach Alpha and Kuder-Richardson Formular-20 Reliability Coefficients

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Abstract - Reliability as one the psychometric properties of a research and measurement instrument is very desirable in research, under the classical test theory (CTT). One group of such reliability measures is that of internal consistency of a scale. Cronbach Alpha and Kuder-Richardson formula 20 are measures of internal consistencies. In many texts, authorities specify that Kuder-Richardson formula 20 should be applied when test items are dichotomously scored while Cronbach Alpha should be applied when test items are not scored dichotomously. These guide lines are religiously adhered to by both researchers and measurement practitioners. Both reliability estimates are recognized as valid measures of internal constituency. In research, an informed choice has to be made about which one to apply, particularly, when only one of them has to be applied. These guidelines are seen as conditions that validate the use of each of them. These specifications are erroneously interpreted. A score is a score whether dichotomous or not. This paper examined both the theoretical and empirical equivalence of these two estimates, for tests of the same length and for varying test lengths. To demonstrate the empirical equivalence of Cronbach Alpha and KR-20, 65 biology achievement tests of the same length (40 items), were administered on 65 samples of senior secondary two students, consisting of 60 students selected from 36 public secondary schools, in Calabar Education zone of Cross River State, Nigeria. Both the Cronbach Alpha and KR-20 were estimated for each test and for the 65 samples. Another test of 100 items adapted from the 2018 and 2019 WAEC-SSCE English Language objective test, was administered on a sample of 300 students. Both reliability measures were estimated for varied lengths of the test, by randomly removing five(5)items at a time. This resulted in 19 sub-tests. The resulting reliability coefficients were compared using the related sample t-test while repeated measures ANOVA was used to compare the reliability coefficients under the influence test length, measured in terms of number of items. The results showed that there is significant difference between Cronbach Alpha and KR-20(t=4.022, p=.000). The correlation between KR-20 and Cronbach alpha is positive and highly significant(r=.944 & .999, p=.000). Test length has significant influence on the difference between KR-20 and Cronbach alpha reliability estimates (F=1372.074, p=.000). The implications of these results for tests developers and users are discussed.

Keywords: Kuder-Richardson formular 20, Cronbach alpha, internal consistency, test length, item discrimination, item difficulty, unidimensionality, principal component analysis.

I. Introduction

Very often educational and psychological researchers as well as practitioners in these areas are requested as a matter of necessity, to estimate the reliability of the instrument they use in data collection. This implies that, during the research process, this property of the measurement instrument has to be estimated. This is usually done by administering well-constructed, face-validated copies of the instrument on a sample, selected from a population of respondents, similar to the population on which the instrument will eventually be administered. This improves the generalizability of the results obtained. The data generated by the trial administration of the instrument are used in estimating the reliability coefficient for the instrument.

Reliability, generally, refers to the extent to which a measuring instrument measures consistently, what it is measuring. However, there are different types of reliabilities or consistencies that may be desirable and they can all be estimated from the data generated from the trial administration of the instrument. When consistency across time is desired, the measure is called across time stability or test-retest reliability. When the extent to which the items of the instrument measure the same trait they are measuring, the reliability is an internal consistency measure. There are many measures of internal consistency. The most common and popularly uses are: split-half, Kuder-Richardson (KR 20 and KR21) and Cronbach
Alpha. The extent to which a rater or scorer is consistent across time is called intra-rater or intra-scorer reliability while the consistency across different raters or scorers is called inter-scorer or inter-rater reliability. Sometimes the interest is on the extent to which an instrument is as good as some other one, the measure is a measure of equivalence, also referred to as parallel form reliability. The computational approaches for each of these measures are different (Umoinyang, et al, 2004; Joshua, 2005; Ebel, 1978; Stanley & Hopkins, 1972).

In practice the most common measures of internal consistency are Cronbach alpha coefficient, Split-half, KR-20 and KR-21. Out of these, Cronbach alpha coefficient and KR-20 are very popular. The choice between these two has been very contentious. Researchers always have the problem of which of them to apply. Some authorities feel that KR-20 should be applied only where test items are measured dichotomously and Cronbach alpha coefficient should be applied where items are not scored dichotomously, e.g. items on Likert scale and essay or short-answer test items. Some others see these specifications as unnecessary, insisting that both KR-20 and Cronbach alpha are the same and should be applied without regards to item format.

However, there are situations where a test consists of both dichotomous and essay type items, measuring the same ability. Sticking to the above conditions will pose a problem. Yet the reliability of such a test must be estimated. An informed choice must be made. This problem gave rise to this study, whose aim was to find out the theoretical and empirical difference (if any) between the Cronbach alpha coefficient and Kuder-Richardson formula 20 reliability estimates; whether the difference between Cronbach alpha and KR-20 is significantly influenced by test length, measured in terms of number of items.

Contributing to the issue of the equivalence of Cronbach alpha coefficient and KR-20, McNeish (2015), Nurachtimah (2018) and Zeynivandnezhad (2021) said that both of them are measures of internal consistency, broadly referred to as coefficient alpha. Under ideal conditions (e.g., no missing data, unidimensionality of items) and with binary items, KR-20 and Cronbach alpha should essentially be the same. If the items are not binary, KR-20, they maintained, will not be appropriate and that Cronbach alpha will be a better choice. This, they continued, would be so particularly in psychological measurement. For Umoinyang et al (2004) KR-20 is a particular case of Cronbach alpha or that Cronbach alpha is the generalization of KR-20. According to Bernard (2018), there seems to be a consensus that internal consistency refers to the extent to which one is measuring a single construct and that this definition does not in any way indicate what construct one may be measuring. He however agreed that KR-20 should be used when dealing with dichotomous item formats. Yeboah (2019) and Qingke-fu (2019) strongly hold the opinion that both Cronbach alpha and KR-20 fit a situation where data are collected on a 5-point Likert scale with responses ranging from strongly agree (SA) to strongly disagree (SD).

A researcher posted a question on https://www.researchgate.net. He sought to know if it was wrong to use Cronbach alpha to assess the internal consistency of a questionnaire with binary options (Yes/No). In his contribution, Emilio (2016) said it is wrong. However, Engel (2020) suggested that different programs should be used and the results compared before a final decision is taken. Wright (2020) said it can be used but suggested that an IRT approach should be adopted, as this is widely used with binary variables, under the single latent trait assumption. Raykov, Dimitrov and Asparouhov (2010) argued that Cronbach alpha cannot be considered in general as a dependable estimator of scale reliability with discrete components. Morgan (2016) maintained that both principal component analysis and Cronbach alpha are based on correlations. So if principal component analysis is considered acceptable to be run on binary data, then, the assumption should apply to Cronbach alpha. This seems to agree with the opinion of Ahmad (2016) that KR-20 is only a special case of Cronbach alpha, when the items are dichotomous. Cronbach alpha, he maintained, is a generalization of KR-20. Recently, he continued, there has been some advocacy for the calculating alpha using tetra-choric correlation instead of phi-coefficient, which is the equivalent of Pearson’s correlation coefficient, whenever the data is binary data. He maintained that calculating Cronbach alpha using Pearson’s correlation yields downward bias results i.e. the calculated alpha is a minimum possible value but that the real or actual reliability may be higher than what is observed. This presumably implies that using tetra-choric correlation yields a better estimate of the reliability. Schermelieh-Engel (2020) in his contribution, added that for binary data, KR-20 is correct. However, he continued, since both KR-20 and Cronbach alpha yield the same results, one can just use Cronbach alpha as provided in the help function of the Statistical Package for Social Sciences (SPSS). He suggested that since KR-20 and Cronbach alpha are based on unidimensionality assumption and tau-equivalence, the difference, if it exists at all, may be tested for significance.

Another researcher raised a question on the web site of Researchgate on what reliability to apply when a test consists of both items that are dichotomous and those that are not. Al-Hemyari (2018) suggested that the first measure of internal consistency to be considered for the proposed instrument should be Cronbach alpha. He maintained that it is worth mentioning that Cronbach alpha is generally among several methods used in psychological and educational studies.
Internal consistency, he continued, refers to the part of a measure that is free from measurement error and can be validly determined by measuring Cronbach alpha, using the “if item is deleted” function in SPSS. He added that it will be helpful to understand that Cronbach alpha is the average value of all possible split-half reliability coefficients. Dogan (2018) added that before using SPSS, it is necessary to ensure that the scale is unidimensional and that items have high positive discrimination indices. He also observed that since SPSS does not use polychoric correlation, the error of measurement in the calculated reliability values may increase, though at an insignificant level. If the sample size and test length, measured in terms of number of items, is large, the results will not change to a level that will change the judgement about reliability.

II. Theoretical equivalence of KR-20 and Cronbach alpha

Cronbach Alpha and Kuder-Richardson 20 as well as CR approaches all yield measures of internal consistency. The Cronbach Alpha approach was developed by Lee Cronbach in 1955 (Umoinyang, et al, 2004). Hesaid for a test of length “n”, consisting of items i=1,2,3…n, the reliability can be estimated using the formula

\[ S_i^2 = \text{the variance of the ith item} \]

\[ S^2 = \text{Scale variance} \]

\[ S_i^2 \text{ is estimated using the scores obtained by all the respondents on the ith item. } S^2 \text{ is estimated using the total scores obtained by each respondent on the scale and for all the respondents. The characteristic specific to this approach is that the items are no dichotomously scored (i.e. 1 for correct response and 0 for wrong response).} \]

As far back as 1937, Kuder and Richardson published the works they had done on the theory of the estimation of test reliability. In that publication (in Ebel, 1978), they presented several approaches to the estimation of reliability, one of which is the Kuder-Richardson formula 20 (KR-20). They said when a test is scored dichotomously (i.e. 1 for correct response and 0 for wrong response) and let the test or scale contain n items; k be the number of correct responses such that the number of wrong responses is N-k, and where N correspond to the number of testees that attempted or responded to the ith item, then KR 20 can be estimated from the formula:

\[ \text{Where } p = k/N \text{ and } q = (N-k)/N \]

\[ n = \text{number of items in the test or scale} \]

\[ i = 1,2,3 \ldots n \]

\[ S_i^2=\text{Scale variance computed based on the total score obtained by each testee or respondent in the test or scale and for all the respondents} \]

\[ k = \text{number of correct responses} \]

\[ \sum(pq) = \text{the sum of the product of } p \text{ and } q \text{ taken over all the items in the test or scale} \]

A careful study of formulas (1) and (2) reveals that n is the same for both formulas as well as \( S_i^2 \). The only difference is \( pq \) and \( S^2 \). So that the only necessary and sufficient condition for equivalence is that \( pq \) is equal to \( S_i^2 \).

Now, if \( p = k/N \) and \( q = (N-k)/N = 1 – k/N \)

Then \( pq = k/N(1-k/N) \)

Secondly, for dichotomously scored items, \( \sum x = \sum x^2 = k \)

Therefore, \( pq = k/N (1 – k/N) \)

\[ = k/N - k^2/N^2 \]

\[ = 1/N(k – k^2/N) \]

Substituting \( \sum x^2 \) and \( \sum x \) for k and \( k^2 \) respectively, we get

\[ pq = 1/N(\sum x^2 – (\sum x)^2/N) \] Q.E.D.

Which is raw score definition of variance.

These results have thus proved that mathematically, Cronbach alpha coefficient and KR-20 are equivalent. The implication of these results is that wherever KR-20 is applicable, Cronbach alpha coefficient can be validly applied. The argument that the Likert scale is continuous while the binary scale in KR-20 is discrete is not true, particularly when these measures are taken as trait measures in IRT. Both scales have underlying continuity and for both scales their zero is not absolute.

The next is whether wherever Cronbach alpha coefficient is applicable, KR-20 can be applied. The evaluation here requires the taking note of the fact that the zero (0) in both cases are not absolute. When a testee gets an item wrong it is taken to mean that he does not know that item. This is not always correct. It seems safer to say that he does not know the item to the point of getting it correct. That means that there is a threshold ability value that if a testee possesses it, he will get the item correct. We can also define a threshold value on a
psychological variable, measured on a Likert scale. If the scale has four response options: Strongly agree (SA), Agree (A), Disagree (D) and Strongly disagree (SD). We can dichotomize this scale by putting all Strongly agree and Agree together and call the response Agree and do the same to Strongly disagree and disagree together as Disagree response. Then for a positively worded statement a response of Disagree can be scored zero (0) and Agree scored one (1). The resulting data becomes a binary data and KR-20 can be applied. The only condition necessary for this equivalence to hold, when the zero on this scale approaches absolute zero, is that the instrument should be validated using principal component analysis in factor analysis (Kerlinger,1986; McNeish, 2015).

From the foregoing review, it is obvious that there is no consensus yet on the equivalence of KR-20 and Cronbach alpha. There is therefore a need to show empirically that:

1) There is no significant difference between KR-20 and Cronbach alpha reliability estimates
2) There is no significant relationship between KR-20 and Cronbach alpha reliability estimates.
3) Test length and reliability type (KR-20 and Cronbach alpha) have no significant influence on the difference between KR-20 and Cronbach reliability estimates.

III. Methodology

Biology multiple choice achievement tests, were developed by 65 post graduate students independently in the faculty of education, Cross River university of technology, Calabar. Each test consisted of 40 items, based on the second term scheme of work for senior secondary two students. The tests were developed as part of their continuous assessment assignment. The test items were vetted by the six lecturers, teaching test and measurement course to the students. This was done to ensure that the tests were face-validated. The tests were administered on a sample of 60 students for each test and in 36 public secondary schools in Calabar Education Zone of Cross River State, Nigeria. As part of the assignment, the post graduate students were required to estimate KR-20 using Microsoft Excel and Cronbach alpha using SPSS version 22.0. Their results were checked for accuracy and correctness by the six lecturers. These reliability estimates were collected for further analysis. The second instrument was a 100-item English Language multiple choice objective test, adapted from the 2018 and 2019 WAEC-SSCE examination question papers. The items were selected to cover the curriculum content of second term of senior secondary two in the 2020/2021 academic session. The items were checked for content validity by three English Language teachers. The test was then administered on a sample of 300 senior secondary two students of a highly populated public secondary school in Calabar. The scores obtained by the students were recorded on an item by person matrix on both Microsoft Excel and SPSS version 22.0. The test length was varied by randomly removing five test items. Each time items were removed, KR-20 was computed from Microsoft Excel and Cronbach alpha from SPSS version22.0. For 65 tests, the KR-20 and Cronbach alpha were compared using dependent (related) sample t-test. The relationship between KR-20 and Cronbach alpha computed using Pearson product moment correlation coefficient. The influences of test length and type of reliability estimate (KR-20 & Cronbach alpha) were tested for significance using repeated measures analysis of variance (ANOVA). All decisions were taken at .05 level of significance.

IV. Results

Descriptive statistics: mean, standard deviation, standard error, were computed for both KR-20 and Cronbach alpha, that were computed for the 65 biology tests.

Hypothesis one

There is no significant difference between KR-20 and Cronbach alpha reliability coefficients

To test this hypothesis, the mean reliability coefficients were then compared using the dependent sample t-test. The results are presented in Table 1.

<table>
<thead>
<tr>
<th>Reliability estimate</th>
<th>Mean difference</th>
<th>Std. Error of difference</th>
<th>t-Value</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>KR-20</td>
<td>.745</td>
<td>.220</td>
<td>.027</td>
<td>.036</td>
</tr>
<tr>
<td>Cronbach alpha</td>
<td>.709</td>
<td>.205</td>
<td>.025</td>
<td>.036</td>
</tr>
</tbody>
</table>

*significant at .05 level, P< .05

The results in Table 1 show that the mean KR-20 and Cronbach alpha are greater than the .700 recommended by Nunnally (1978). This difference was, however, not tested for significance, because such tests fell outside the scope of this study. The p-value (.000) associated with the computed t-value (4.022) is less than .05, the chosen level of significance. Thus the null hypothesis was rejected in favour of the alternative. This means that there is significant difference between the computed mean KR-20 and Cronbach reliability coefficients, with the mean KR-20 being greater than Cronbach alpha.
Hypothesis two

There is no significant relationship between KR-20 and Cronbach alpha reliability coefficients.

To test this hypothesis, Pearson product moment correlation coefficient was applied. The results show that correlation coefficient between KR-20 and Cronbach reliability coefficients, is positive (.944) and significant (p=.000, p < .05). This means that there is a significantly high and positive relationship between KR-20 and Cronbach reliability coefficients.

Hypothesis three

Test length and reliability type (KR-20 and Cronbach alpha) have no significant influence on the difference between KR-20 and Cronbach reliability estimates.

The Pearson product moment correlation coefficient was computed for all possible pairs of the three variables in the hypothesis-test length, KR-20 and Cronbach alpha, together with their associated p-values. The results are presented in Table 2.

Table 2: Inter-variable correlation among test length, KR20 and Cronbach alpha

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Test length</th>
<th>Cronbach alpha</th>
<th>KR-20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test length</td>
<td>1**</td>
<td>.584*</td>
<td>.578*</td>
</tr>
<tr>
<td>Cronbach alpha</td>
<td>.009</td>
<td>1</td>
<td>.999*</td>
</tr>
<tr>
<td>KR-20</td>
<td>.010</td>
<td>.000</td>
<td>1</td>
</tr>
</tbody>
</table>

*significant at .05 level. **Values above the main diagonal are correlation coefficient, and those below it, are corresponding p-values.

The results in Table 2 show that all the correlation coefficients are positive (.584 < p < .999) and significant. This means that both KR-20 and Cronbach alpha increase significantly with increasing test length. The correlation between KR-20 and Cronbach alpha is positive and very high (.999) near perfect.

To test the hypothesis, repeated measures analysis of variance (ANOVA) was applied, test length (measured in terms of number of items) and reliability type (KR-20 and Cronbach alpha) as factors and reliability coefficient as the dependent variable while Fisher’s F-ratio test was used to test for the significance of the influence. The results obtained are presented in Table 3.

Table 3: Repeated measures ANOVA of reliability estimates by test length and reliability type

<table>
<thead>
<tr>
<th>Rel. Coeff.</th>
<th>N</th>
<th>Mean</th>
<th>Std.dev.</th>
<th>Std.Error</th>
<th>Mean Difference</th>
<th>Std.dev. of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cron. alpha</td>
<td>19</td>
<td>7592</td>
<td>.0916</td>
<td>.0114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KR-20</td>
<td>19</td>
<td>7671</td>
<td>.0931</td>
<td>.0115</td>
<td>-.0079</td>
<td>.005</td>
</tr>
<tr>
<td>Source of variation</td>
<td>Sum of squares</td>
<td>df</td>
<td>Mean squares</td>
<td>F-value</td>
<td>p-value</td>
<td>Remarks</td>
</tr>
<tr>
<td>Corr. model</td>
<td>.3080</td>
<td>37</td>
<td>.008</td>
<td>334.728*</td>
<td>.000</td>
<td>significant</td>
</tr>
<tr>
<td>Intercept</td>
<td>1.0110</td>
<td>1</td>
<td>1.011</td>
<td>81598.063*</td>
<td>.000</td>
<td>**</td>
</tr>
<tr>
<td>Test length</td>
<td>.3070</td>
<td>18</td>
<td>.017</td>
<td>1372.074*</td>
<td>.000</td>
<td>**</td>
</tr>
<tr>
<td>Rel. type</td>
<td>.0010</td>
<td>1</td>
<td>.001</td>
<td>80.710*</td>
<td>.000</td>
<td>**</td>
</tr>
<tr>
<td>Error</td>
<td>.0002</td>
<td>18</td>
<td>1.239E-5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.3190</td>
<td>38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr. total</td>
<td>.3080</td>
<td>37</td>
<td></td>
<td>R = .999</td>
<td>R2= .998</td>
<td></td>
</tr>
</tbody>
</table>

Significant at .05 level. P< .05

The results in Table 3 show that the mean KR-20 (.7671) is greater than the mean Cronbach alpha (.7592) and the pattern for their standard deviation is the same. This explains why the mean is -.0079. The p-values (.000) associated with the computed F-values (334.728, 81598.063, 1372.074 & 80.710) the corrected model, intercept, test length, and reliability type, are all less than .05 the chosen level of significance. The null hypothesis relating to the contribution of these components of the ANOVA model, make significant contribution in the model. Specifically, the influence of test length and reliability type on reliability estimates, are significant. That means there is a significant difference between KR-20 and Cronbach alpha. These results also show that the difference between KR-20 and Cronbach alpha for differing test length is not constant.

V. Discussion

The results that showed that the difference between KR-20 and Cronbach reliability estimates is significant, was not expected at all. What was expected was a non-significant difference, a difference that would have been attributed to rounding-up errors. The results however confirmed the fears expressed by McNeish (2015) and Nurchotimah(2018) that when the test is not unidimensional, Cronbach alpha consistently underestimates reliability, providing the lower bound of the reliability estimates. This was observed for the situation where the test length is constant and when the test length was varied. Since in both cases the tests were not validated for unidimensionality, the results are consistent. The most hard warming results is that in 54 out of the 65 pairs, the
reliability estimates were the same up to the second decimal place. Differences were observed only in the third decimal place. It seems reasonable to agree with McNeish (2015), Morgan (2016), Ahmad (2016), Nurchotimah (2018), Bernard (2018) and Raykov, Dimitrov and Asparouhov (2010), that if the test is validated and under the latent trait assumption, Cronbach alpha can be validly applied. So that emphasis should shift from whether the test is scored dichotomously to validity that more reasonable.

The results also showed that there is a very strong positive (.944 & .999), between KR-20 and Cronbach alpha, for tests of same length and varied test length, respectively. This agrees with the opinion of Dogan (2018) if the sample size and test length, measured in terms of number of items, are large enough, the results will not differ to a level that will alter the judgement about the reliability. This has also been validated by the fact that the influence of test length in this study is highly significant as well as previous results reported by Uyanah (2007). The high correlation coefficient (.999) recorded for the longer test developed by WAEC can be attributed to the fact WAEC takes a lot of care in the development of its test items.

VI. Conclusion

From the results of this study, it is hereby concluded that KR-20 and Cronbach alpha are both theoretically and numerically equivalent when the test items are dichotomously scored and the number of items in the test. This equivalence improves with increasing test length.

VII. Recommendations

The Cronbach alpha reliability coefficient should be use for estimating reliability wherever KR-20 is applicable. Test developers should pay more attention to validity issues instead of the current emphasis on whether the test is scored dichotomously or not. It may be necessary ti find out if dichotomizing the scores on a scale like the Likert scale and applying KR-20 will significantly affect the value of the reliability estimates.

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Citation of this Article:

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