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Applications of General Integral Transform for Solving Mathematical Models Occurring in the Environmental Science and Biotechnology

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***Abstract* - Lot of Mathematical Models including differential equations plays an important role in healthcare and biotechnology. One of these is in Malthus model. This model was developed by Thomas Malthus, in his essay on world Population Growth and resource supply. Another interesting equation is Advection diffusion equation and predator prey model. We use a integral transform called General Integral Transform to obtain the solutions of these models which are important in biotechnology and health sciences.**

Keywords: General integral transform; Mathematical modelling; Predaror Prey model; Malthus model.

1. Introduction

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms. Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Newton's law of Cooling is solved by using Kushare transform [3], Emad- Falih transform [22], Soham transform [23], HY integral transform[25] In April 2022 D. P. Patil, et al [4] used Kushare transform, HY integral transform[24], Emad Sara transform [28], Alenzi transform [29], Emad Falih transform[30], Kharrat Toma transform [39], KKAT transform[43], ARA transform[47], Rangaig integral transform[48] and KKA transform [51] for solving the problems on population growth and decay. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9]. D .P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. The system of differential equations using Kushare transform[12] and Emad Sara transform [55]. D. P. Patil [13] used Aboodh and Mahgoub transforms to solve boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems. Laplace, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al obtained solution of Volterra Integral equations of first kind by using, Anuj transform [17], Kushare transform [34]. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. [19] Emad Sara transform[19] and Emad-Falih transform [26] are used for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil et al [27] introduced double kushare transform. Nikam, Patil et al [31] used, Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, et al [36] developed generalized double rangaig integral transform. Shinde, et al [37] used Kushare transform is used for solving Volterra Integro-Differential equations of first kind. Patil et al [38] used new general integral transform [49] and Kushare transform [52] to solve Abel's integral equations. Patil et al

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[40] used Kushare transform for evaluating integrals containing Bessel's functions. Thakare and Patil [41] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [42]. Models in health sciences and biotechnology are solved by using Soham transform [44] and Kushare transform [50]. Kushare transform and NE transform is used in mechanics [46], [53]. KAJ transform is used to solve stochastic differential equations [56].

We organize this paper as follows. Introduction is included in section one. Second section is devoted for preliminary concepts. General integral transform is used to the logistic growth model in in section three and predetor Prey model in fourth section. Applications are in fifth section. Section six is for conclusion and seven for acknowledgement.

2. Preliminary

In this section we include some required definitions; some useful formulae and theorems based on general integral transform.

Definition: The general integral transform of the function $f(t)$ is defined for $t \geq 0$, $p(s) \neq 0$ and q(s) are positive real function we define the general integral transform *T(s)* of f(t) by the formula

$$T\{f(t);s\} = T(s) = p(s)\int_0^\infty f(t)e^{-q(s)}dt$$

Table No. 1: General Iintegral Transform of Some Standard functions

Sr.No.	f(t)=$T^{-1}\{T(s)\}$	T(s)=T{f(t);s}
1.	1	$\frac{p(s)}{q(s)}$
2.	t	$\frac{p(s)}{q^2(s)}$
3.	t^α	$\frac{T[\alpha+1]p(s)}{q(s)^{\alpha+1}}, \alpha > 0$
4.	$\sin(t)$	$\frac{p(s)}{q^2(s)+1}$
5.	$\sin(at)$	$\frac{ap(s)}{a^2+q^2(s)}, if\ q(s) > 1$
6.	$\cos(t)$	$\frac{q(s)p(s)}{q^2(s)+1}$
7.	e^t	$\frac{p(s)}{q(S)-1}$
8.	$tH(t-1)$	$\frac{e^{-q(s)}(q(s)+1)p(s)}{q^2(S)}$
9.	$f'(t)$	$q(s)T(s) - p(s)f(0)$

General integral transform of Derivatives: If f(t) is a function of exponential order then

$$T\{f'(t);s\} = q(s)T(s) - p(s)f(0)$$

3. General integral transform for Logistic Growth Model

Consider the Logistic growth model equation

$$\frac{du}{dt} = u - (f(u), \; u(0) = u_0 \quad ....(3.1)$$

Here f is non-linear function of u and suppose that solution u of equation (3.1) is of infinite power series as follows,

$$u = u(t) = \sum_{n=0}^{\infty} a_n t^n \; ....(3.2)$$

Further (3.2) also satisfies the conditions for existence of General integral Transform.

Applying General integral transform on both sides of (3.1) we get,

$$T\left(\frac{du}{dt}\right) = T(u) - Tf(u)$$

$$q(s)T(s) - p(s)f(0) = T(s) - F(s)$$

Where *T*(s) =T(u(t)) and F(s) =T(f(u)) are the general integral transform of the functions u(t) and f(u) respectively.

Rearranging the terms in (3.3) we get,

$$T(s) = u_0 \frac{p(s)}{q(s)-1} - \frac{F(s)}{q(s)-1} \quad .... (3.4)$$

If we suppose (u) $=$ u^2 then

$$f(u) = \left(\sum_{n=0}^{\infty} a_n t^n\right)^2$$

$$=(a_o + a_1 t + a_2 t^2 + a_3 t^3 + ... + a_n t^n)^2$$

$$={a_0}^2 + 2a_0 a_1 t + (2a_0 a_2 + {a_1}^2)t^2 + (2a_0 a_3 +$$
2a1a2t3+ …….. .... (3.5)

Taking General integral transform on both sides of (3.5),

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$$F(s) = (a_o)^2 \frac{p(s)}{q(S)} + 2a_0 a_1 \frac{p(S)}{q^2(s)} + (2a_0 a_2 + {a_1}^2)\frac{2!\,p(s)}{q^3(S)} + (2a_0 a_3 + 2a_1 a_2)\frac{3!p(s)}{q^4(s)} + \ldots\ldots$$

$$\therefore \frac{F(s)}{q(s)-1} = \frac{{a_o}^2 p(s)}{(q(s)-1)q(s)} + \frac{2a_o\, a_1 p(s)}{(q(s)-1)q^2(s)} + \frac{2!\,(2a_o\, a_2 + a_1^2)p(s)}{(q(s)-1)q^3(s)} + \frac{3!(2a_o\, a_3 + 2a_1 a_2)p(S)}{(q(s)-1)q^4(s)} + \ldots.$$

Applying partial fractions on R.H.S Of the above equation,

$$\frac{F(s)}{q(s)-1} = e^t\left[{a_0}^2 + 2a_0 a_1 + 4a_0 a_2 + 2{a_1}^2 + 12a_0 a_3 + 12a_1 a_2 + \ldots\right] - \left[{a_0}^2 + 2a_0 a_1 + 4a_0 a_2 + 2{a_1}^2 + 12a_0 a_3 + 12a_1 a_2 + \ldots\right] - t\left[2a_0 a_1 + 4a_0 a_2 + 2{a_1}^2 + 12a_0 a_3 + 12a_1 a_2 + \ldots\right]$$

$$-\frac{t^2}{2!}\left[4a_0 a_2 + 2{a_1}^2 + 12a_0 a_3 + 12a_1 a_2 + \ldots\right] - \frac{t^3}{3!}\left[12a_0 a_3 + 12a_1 a_2 + \ldots\right]\ldots\ldots\ldots.$$

Applying inverse general integral transform to both sides of the above equation and rearranging the terms we get,

$$u(t) = u_0\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots..\right) - \left[a_0^2 t + ({a_0}^2 + 2a_o\, a_1)\frac{t^2}{2!} + ({a_0}^2 + 2a_0 a_1 + 4a_0 a_2 + 2{a_1}^2)\frac{t^3}{3!} + \cdots\right]$$

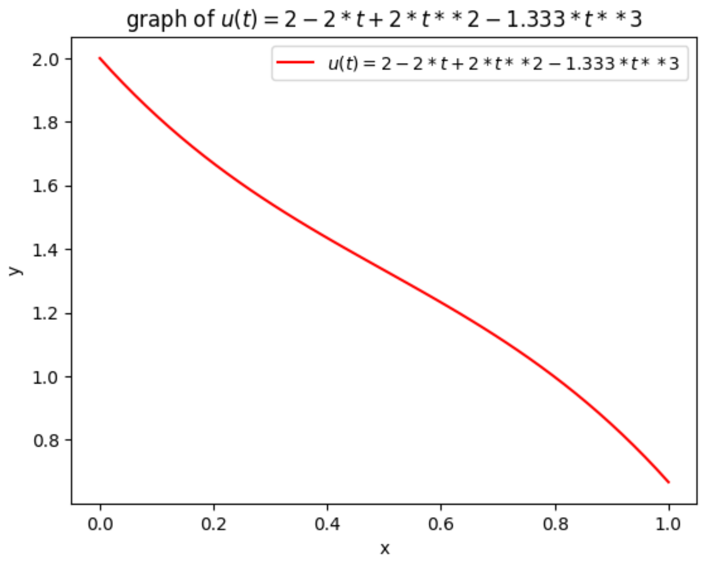
$$\therefore u(t) = u_0 + (u_0 - a_0^2)t + (u_0 - {a_0}^2 - 2a_0 a_1)\frac{t^2}{2!} + (u_0 - {a_0}^2 - 2a_0 a_1 - 4a_0 a_2 - 2{a_1}^2)\frac{t^3}{3!}$$

If we consider $u_0 = 2$ and compare with (3.2) we obtain

$$a_o = 2, \quad a_1 = -2, a_2 = 2, a_3 = -\frac{4}{3}$$

$$\therefore u(t) = 2 - 2t + 2t^2 - \frac{4}{3}t^3\ldots..$$

It is required solution and the graph of this solution is



This figure 1 is a graph showing that how the population of a species changes when a hazard function is acting in life of the species.

From the graph, we can conclude that, if there is more competition in life and more hazards then the population decreases. Here f(u) that is hazard function is taken as square of the population that means more hazard , so population of insects reaches to zero in less than one unit interval of time.

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4. General Integral Transform for Predator Prey Model

In this section we use General Integral Transform for Predator Prey Model. The interaction between two species and their effect on each other is called as predator Prey relationship. In this one species is feeding on the other species. An organism that eats or hunts other organism as food is called as predator and an organism that is killed by other organism for food is called as prey. Fox and Rabbit, Lion and Zebra are examples of predator and prey. This concept of predator prey is not only applicable for animals but it is applicable for plants too. Grasshopper and Leaf is an example of this.

Consider the system of differential equations governing predator prey model,

$$\frac{du}{dt} = u - f(u,v) \qquad \ldots\ldots. (4.1)$$

$$\frac{dv}{dt} = \beta[g(u,v) - v] \qquad \ldots\ldots. (4.2)$$

With initial conditions u(0) = u_0 and v(0) = v_0, f and g are nonlinear functions of u and v. β is a positive constant. Let u and v be the solutions of this system, which are infinite series of the form $u = (t) = \sum_{n=0}^{\infty} a_n t^n, v = \sum_{n=0}^{\infty} b_n t^n$ and they both also satisfy the required conditions for existence of General integral Transform.

Applying General integral Transform to both sides of (4.1) and (4.2),

$$\mathrm{T}\left(\frac{du}{dt}\right) = T(u) - T(f(u,v))$$

$$\mathrm{T}\left(\frac{dv}{dt}\right) = \beta[T[g(u,v)] - T(v)]$$

Using the General integral transform of Derivative theorem

$$q(s)T(\mathrm{s}) - \mathrm{p(s)u(0)} = \mathrm{U}(s) - \mathrm{F}(s)$$

$$q(s)T(\mathrm{s}) - \mathrm{p(s)v(0)} = \beta\mathrm{G}(s) - \beta\mathrm{V(s)}$$

Where $\mathrm{T(u) = U(s),\ T(s) = V(s), T[f(u(t), s(t))] = F(s)}$ and $\mathrm{T[g(u(t), s(t))] = G(s)}$

Rearranging the terms and simplifying we get,

$$\mathrm{U(s)} = u_0 \frac{p(s)}{q(s) - 1} - \frac{F(s)}{q(s) - 1}$$

$$\mathrm{V(s)} = V_0 \frac{p(s)}{q(s) + \beta} + \frac{\beta G(s)}{q(s) + \beta}$$

Applying inverse General integral Transform

$$\mathrm{u(t)} = u_0 e^t - T^{-1}\left(\frac{F(s)}{q(s)-1}\right) \ldots\ldots.. (4.3)$$

$$\mathrm{v(t)} = v_0 e^{\beta t} + \beta T^{-1}\left(\frac{G(s)}{q(s)+\beta}\right) \ldots\ldots... (4.4)$$

Equations (4.3) and (4.4) represent the solution of system of equations (4.1) and (4.2),

5. Applications and Results

In this section we use results in above section to solve some systems of differential equations arising in biotechnology and health of sciences.

Example 1: Consider the system of differential equations governing predator prey model.

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$\frac{du}{dt} = u - uv$ ...... (5.1)

$\frac{dv}{dt} = uv - v$ ...... (5.2)

With initial conditions $u(0) = 1.3, v(0) = 0.6$

Suppose,

$u = \sum_{n=0}^{\infty} a_n t^n , v = \sum_{n=0}^{\infty} b_n t^n$ be the solutions of the system (5.1) and (5.2).

$$uv = a_0 b_0 + (a_0 b_1 + a_1 b_0)\mathrm{t} + (a_0 b_2 + a_1 b_1 + a_2 b_0) t^2 + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) t^3 + \ldots.$$

Applying General integral Transform,

$$\therefore \mathrm{T}(uv) = a_0 b_0 \frac{p(s)}{q(S)} + (a_0 b_1 + a_1 b_0) \frac{p(S)}{q^2(s)} + (a_0 b_2 + a_1 b_1 + a_2 b_0) \frac{2!\, p(s)}{q^3(S)} + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) \frac{3!\, p(s)}{q^4(s)} + \ldots..$$

Suppose, $T(uv) = F(s) = G(s)$

By previous section we have,

$$T(s) = u_0 \frac{p(s)}{q(s) - 1} - \frac{F(s)}{q(s) - 1}$$

$$T(s) = 1.3 \frac{p(s)}{q(s) - 1} - \left[\frac{a_0 b_0}{(q(s) - 1)} \frac{p(s)}{q(S)} + \frac{(a_0 b_2 + a_1 b_1 + a_2 b_0)}{(q(s) - 1)} \frac{p(S)}{q^2(s)} + \frac{2(a_0 b_2 + a_1 b_1 + a_2 b_0)}{(q(s) - 1)} \frac{2!\, p(s)}{q^3(S)} + \frac{6(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)}{(q(s) - 1)} \frac{3!\, p(s)}{q^4(s)} + \ldots.. \right]$$

Rearranging the terms,

$$T(s) = 1.3 \frac{p(s)}{q(s) - 1} - \left[\frac{a_0 b_0}{(q(s) - 1)} \frac{p(s)}{q(S)} \frac{q(s)}{q(s)} + \frac{(a_0 b_2 + a_1 b_1 + a_2 b_0)}{(q(s) - 1)} \frac{p(S)}{q^2(s)} \frac{q^2(S)}{q^2(S)} + \frac{2(a_0 b_2 + a_1 b_1 + a_2 b_0)}{(q(s) - 1)} \frac{2!\, p(s)}{q^3(S)} \frac{q^3(S)}{q^3(S)} + \frac{6(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)}{(q(s) - 1)} \frac{3!\, p(s)}{q^4(s)} \frac{q^4(S)}{q^4(S)} + \ldots.. \right]$$

Applying partial fractions and Rearranging the terms

$$T(s) = 1.3 \frac{p(s)}{q(s) - 1} - [a_0 b_0 \left(\frac{1}{(q(s) - 1)} - 1 \right) + (a_0 b_1 + a_1 b_0) \left[-1 - \frac{1}{q(s)} + \frac{1}{(q(s) - 1)} \right] + 2\,(a_0 b_2 + a_1 b_1 + a_2 b_0) \left[-1 - \frac{1}{q(s)} - \frac{1}{q^2(s)} + \frac{1}{(q(s) - 1)} \right] + 6\,(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) \left[-\frac{1}{q(s)} - \frac{1}{q^2(s)} - \frac{1}{q^3(s)} \right] + \ldots..$$

Applying inverse General integral Transform,

$$u(t) = 1.3 + (1.3 - a_0 b_0) t + (1.3 - a_0 b_0 - a_0 b_1 - a_1 b_0) \frac{t^2}{2!} + (1.3 - a_0 b_0 - a_0 b_1 - a_1 b_0 - 2 a_0 b_2 - 2 a_1 b_1 - 2 a_2 b_0) \frac{t^3}{3!}$$

Similarly, we can obtain,

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$v(t) = 0.6 + (0.6 - a_0b_0)t + (0.6 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (-0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \cdots)$ ……. (5.4)

From equation (5.3)

$$\sum_{n=0}^{\infty} a_n t^n = 1.3 + (1.3 - a_0b_0)t + (1.3 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \ldots$$

Hence, $a_0 = 1.3$, $a_1 = 1.3 - a_0b_0$, $a_2 = 1.3 - a_0b_0 - a_0b_1 - a_1b_0$, $a_3 = 1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0$, … …

From equation (5.4)

$$\sum_{n=0}^{\infty} b_n t^n = 0.6 + (0.6 - a_0b_0)t + (0.6 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (-0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \ldots$$

Hence, $b_0 = 0.6$, $b_1 = (0.6 - a_0b_0)$, $b_2 = (0.6 - a_0b_0 - a_0b_1 - a_1b_0)$, $b_3 = (-0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)$, ……

Obtaining values of $a_0, a_1, a_2, a_3, \ldots, b_0, b_1, b_2, b_3, \ldots$

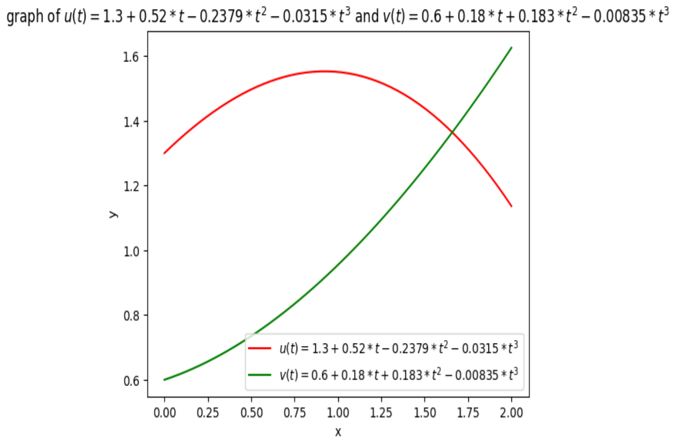
We get required solution of the system of equations

$$u(t) = 1.3 + 0.52\,t - 0.2379t^2 - 0.03153t^3 + \cdots$$

and

$$v(t) = 0.6 + 0.18t + 0.183t^2 - 0.00835t^3 + \cdots \ldots$$

Graph of the system (5.1) and (5.2) with given initial conditions is,



This graph is showing effect of predators on preys.

From this graph we can conclude that the number of predators and prey is maintained (conserved) in some limit. That means if the number of prey increases then the number of predators will also increases due to increase in food supply. Increase in the predators consumes more food. It results reduction in food supply which means number of prey reduces. A time comes when the number of predators and prey becomes equal. Then increase in predator results decrease in prey. Hence there is shortage of food for predators. Thus the chain is continued and number of predators and prey always remains in some specific limit.

6. Conclusion

By using General integral transform we can easily solve the mathematical models in biochemistry, health sciences, and environmental sciences, containing ordinary differential equations.

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