International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048
Volume 7, Issue 12, pp 189-202, December-2023
https://doi.org/10.47001/IRJIET/2023.712027

A Generalized Efficient Family of Ratio Estimator of Population Mean Under Double Sampling Techniques

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***Abstract* - In continuous search for a better method of estimating the population mean in double sampling techniques, with advantageous properties than some existing ones and to address the problem of efficiency thereon, a generalized efficient family of ratio estimator of population mean under double sampling techniques is suggested using information on a single supplementary variable. Members of the suggested family of estimator were obtained by modifying the values of the scalars associated with the suggested estimator. These estimator and its members were then transformed to an expanded form, from where their properties such as Biases, relative biases, Mean Square Errors (MSEs), and Optimal Mean Square Errors (OMSEs) were derived to the second order of Taylor series approximation. Theoretical underpinnings and conditions for measuring efficiency of an estimator over its competitor estimators were established and validated with some natural population data sets. The results showed that the generalized efficient family of ratio estimators in double sampling techniques produced smaller MSEs which is an indicator of appreciable gain in efficiency and dominance over some existing ratio estimators and were therefore deemed to provide a better substitute whenever priority is placed on efficiency.**

Keywords: MSEs, Double Sampling, Efficiency, Estimator and Population Mean.

1. Introduction

It is a fact that sample survey became imperative to savage financial inadequacy, limited time, manpower, scarcity of units and other associated risks of studying the entire population by method of complete enumeration or census procedure. Statisticians over the years have been unrelenting towards achieving the main goal of statistical survey which is to reduce sampling errors either by devising suitable sampling techniques or by formulating efficient estimator of the population parameters.

In sampling procedure, estimation of population parameters has been of paramount interest. This estimation most times seek to make use of better methods of estimation that would give an improved result. Most often, interest has been on the mean of a certain characteristic of a finite population on the basis of the sample taken from the population following a specific sampling procedure. Since the mean has much application in sampling and statistical analysis.

The ratio and regression methods of estimation require the knowledge of population mean of supplementary variable $\bar{X}$to estimate the population mean of study variable$\bar{Y}$. When such information on the supplementary variable is not available or feasible, it is sometimes cheap to take a large preliminary sample in which $x_{i's}$ alone is measured. The purpose of this sample is to furnish a good estimate of $\bar{X}$or of the frequency distribution ofx_i. In some survey whose function is to make estimate of some other variate y_i , it may pay to devote part of the resources to this preliminary sample. This techniques is known as double sampling or two phase sampling.

Literatures on sampling is quite vast and traceable to the early part of the 20th century to[3] who laid the foundational stone of modern sampling dealing with stratification, [15], [16],advocated two-phase sampling to address the problem of strata sizes in stratified sampling, while the estimation of population mean in double sampling for the classical ratio estimator of [7],was first advocated by [23]. Other authors who suggested variety of ratio estimators in double sampling scheme included; [11], [20], [19], [21], [22], and [4], to mention but a few.

The search for a better estimator of population mean that will give an improved result in double sampling techniques and in a view to provide a better alternatives to the existing ratio estimators made many authors to propose variety of estimators' which were found to give an improved results under certain conditions. Notably among them were; [10], [12], [18], [5], [22], [21], [5], [8],[9], [17],[2], [1] and [25], etcetera.

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Volume 7, Issue 12, pp 189-202, December-2023

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In continuous search for a better method of estimating the population means in double sampling scheme with desirable properties than some existing ones and to address the problem of efficiency thereon, this study puts forward a generalized efficient family of ratio estimator of population mean under double sampling techniques using information on a single supplementary variable.

2. Methodology

This study applies the method of mathematical expectation and Taylor's series approximation to the second order to derive the theoretical results. Some existing estimators (with their properties) that are related to this study are presented. A generalized efficient family of ratio estimator of population mean in double sampling is suggested.

The properties of the suggested estimator as well as its optimality condition were obtained. This condition was then used to obtain an expression for the Asymptotic Optimal Estimator (AOE), its bias and MSE.

2.1 Sampling method

(a) Simple Random Sampling description

Let $\{U_1, U_2, \dots, U_N\}$ be a finite population having N units, where U_i is a pair of values (X_i, Y_i), $i = 1, 2, \dots, N$, Here Y is a study variable and X is the supplementary variable which is correlated with Y. Let $(y_1, y_2, \dots, y_n)$ and $(x_1, x_2, \dots, x_n)$ be n sample values, then under Simple Random Sampling Without Replacement (SRSWOR), the means and variances of the study and supplementary variables are given as:

$$\left.\begin{aligned} \bar{X}_{SRS} &= \frac{1}{N}\sum_{i=1}^{M} X_i \\ \bar{Y}_{SRS} &= \frac{1}{N}\sum_{i=1}^{M} Y_i \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} \mathrm{Var}(\bar{X}_{SRS}) &= \left(\frac{1-f}{n}\right)\sigma_x^2 \\ \mathrm{Var}(\bar{Y}_{SRS}) &= \left(\frac{1-f}{n}\right)\sigma_y^2 \end{aligned}\right\} \qquad (2)$$

If the finite population correction $f \neq 0$

$$\left.\begin{aligned} \sigma_x^2 &= (N-1)^{-1}\textstyle\sum_{i=1}^{N}(X_i-\mu_x)^2 \\ \sigma_y^2 &= (N-1)^{-1}\textstyle\sum_{i=1}^{N}(Y_i-\mu_y)^2 \\ \sigma_{xy}^2 &= (N-1)^{-1}\textstyle\sum_{i=1}^{N}(X_i-\mu_x)(Y_i-\mu_y) \end{aligned}\right\} \qquad (3)$$

$$\left.\begin{aligned} R &= \frac{\mu_y}{\mu_x} \\ \rho_{xy} &= \frac{\sigma_{xy}}{\sigma_x\sigma_y} \\ \sigma_{xy} &= Cov(x,y) \\ C_y^2 &= \frac{\sigma_y^2}{\mu_y^2} \\ C_x^2 &= \frac{\sigma_x^2}{\mu_x^2} \end{aligned}\right\} \qquad (4)$$

2.2 Double sampling description

Let $\pi_i = \{\pi_1, \pi_2, \pi_3, \cdots \pi_N\}$ be a population containing the study and supplementary variate taking values on the π_i. Two approaches or cases of estimating the population mean are presented below:

Case I: "A large preliminary sample of size n_1 is selected by simple random sampling Without replacement (SRSWOR) from the population of N units and information is obtained on the supplementary variable alone. A second sub-sample of size $n_2, (n_2 < n_1)$ is selected by simple random sampling SRSWOR. Information on Y is obtained from the second phase sub-sample".

Case II: A second sample of size n_2 is obtained from the population independent of the first phase sample and information on both the supplementary and study character are obtained from this sample.

Some existing simple random sampling ratio estimators of the product mean μ_y of the study variable Y alongside their MSE's are presented in table 1: where $f = \frac{n}{N}$, N is the population size, n is the sample size, R is the population ratio, ρ_{xy} is the correlation coefficient between X and Y, σ_{xy} is the covariance between X and Y, $\lambda = \frac{1}{n_2} - \frac{1}{N}$ and $\lambda' = \frac{1}{n_1} - \frac{1}{N}$

Table 1: Some existing ratio estimators in double sampling with MSE

S/N	Estimators	MSE
1.	$\bar{y}$, Sample Mean	$\lambda\bar{Y}^2 C_y^2$
2.	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$, Sukhatme (1962)	$\bar{Y}^2[\lambda C_y^2 + (\lambda - \lambda')C_x^2(1-2k)]$
3.	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^\alpha$, Srivastava (1970)	$\bar{Y}^2\{\lambda C_y^2 + (\lambda - \lambda')\alpha C_x^2(\alpha - 2k)\}$
4.	$\bar{y}\exp\left[\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right]$, Singh & Vishwakarma (2007)	$\bar{Y}^2\{\lambda C_y^2 + \frac{(\lambda - \lambda')}{4}C_x^2(1-4k)\}$
5	$\bar{y}\exp\delta\left[\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right]$, Singh et al (2014)	$\bar{Y}^2\{\lambda C_y^2 + \delta\frac{(\lambda - \lambda')}{4}C_x^2(\delta - 4k)\}$
6	$\bar{y}\left[\alpha\left(\frac{\bar{x}_2}{\bar{X}}\right) + (1-\alpha)\left(\frac{\bar{X}}{\bar{x}_1}\right)\right]$, Singh & Choudhury (2012)	$\bar{Y}^2\lambda C_y^2(1-\rho^2)$
7.	$\bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$ Bahl and Tuteja (1991)	$\frac{\bar{Y}^2}{n}\{C_y^2 + \frac{C_x^2}{4} - 4\rho C_x C_y)\}$
8	$\bar{y}\exp\left[\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right]$ Bahl and Tuteja (1991)	$\frac{\bar{Y}^2}{n}\{C_y^2 + \frac{C_x^2}{4} + 4\rho C_x C_y)\}$
9	$\bar{y}\left[\frac{\bar{x}_1 + a\bar{x}_2}{a\bar{x}_1 + \bar{x}_2}\right]$ Etorti (2019)	$\bar{Y}^2 C_y^2\left(\lambda' + \left(\frac{1}{n_2} - \frac{1}{n_1}\right)(1-\rho^2)\right)$

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2.3 The suggested generalized efficient family of ratio estimator for population mean under double sampling techniques

Motivated by [8] ratio estimator of population mean under double sampling scheme, a generalized efficient family of ratio estimator for population mean under double sampling techniques is suggested and presented as follows:

$$t_{drg} = \bar{y}\left(\frac{\bar{x}_1 + a^*\bar{x}_2}{a^*\bar{x}_1 + \bar{x}_2}\right)^{\gamma} \qquad (5)$$

Where

$\bar{y} = \sum_{i=1}^{n}\frac{y_i}{n}$, the sample mean of the variable of interest obtained from the second phase sample

$\bar{x}_1 = \sum_{i=1}^{n_1}\frac{x_{1i}}{n_1}$, the first phase sample mean of the supplementary variable

$\bar{x}_2 = \sum_{i=1}^{n_2}\frac{x_{2i}}{n_2}$, the second phase sample mean of the supplementary variable

$\bar{Y} = \sum_{i=1}^{N}\frac{Y_i}{N}$, the unknown population mean of the study variable Y

$\bar{X} = \sum_{i=1}^{N}\frac{X_i}{N}$, the unknown population mean of the supplementary variable X

a^* and γ are suitably chosen scalars.

Letting

$$\left.\begin{aligned} e_y = \frac{\bar{y}-\bar{Y}}{\bar{Y}},\ e_{x_1} &= \frac{\bar{x}_1-\bar{X}}{\bar{X}},\ e_{x_2} = \frac{\bar{x}_2-\bar{X}}{\bar{X}} \\ \bar{x}_1 &= \bar{X}(1+e_{x_1}) \\ \bar{x}_2 &= \bar{X}(1+e_{x_2}) \\ \bar{y} &= \bar{Y}(1+e_y) \end{aligned}\right\} \qquad (6)$$

(5), can be transformed as;

$$t_{drg} = \bar{Y}(1+e_y)\left(\frac{1+e_{x_1}+a^*+a^*e_{x_2}}{a^*+a^*e_{x_1}+1+e_{x_2}}\right)^{\gamma}$$

$$= \bar{Y}(1+e_y)\left(\frac{(1+a^*)\left[1+\frac{e_{x_1}}{(1+a^*)}+\frac{a^*e_{x_2}}{(1+a^*)}\right]}{(a^*+1)\left[1+\frac{a^*e_{x_1}}{(1+a^*)}+\frac{e_{x_2}}{(1+a^*)}\right]}\right)^{\gamma}$$

$$= \bar{Y}(1+e_y)(1+u)^{\gamma}(1+v)^{-\gamma} \qquad (7)$$

Where $u = g_1e_{x_1} + g_2e_{x_2}$, $v = g_2e_{x_1} + g_1e_{x_2}$, $g_1 = \frac{1}{1+a^*}$, $g_2 = \frac{a^*}{1+a^*}$

Assuming $|v| < 1$, $(1+v)^{-\gamma}$ can be expanded using Taylor's approximation as

$$t_{drg} = \bar{Y}(1+e_y)(1+u)^{\gamma}\left(1-\gamma v + \gamma\left(\frac{\gamma+1}{2}\right)v^2+\cdots\right)$$

$$= \bar{Y}(1+e_y)\left[1+\gamma u+\gamma\left(\frac{\gamma-1}{2}\right)u^2+\cdots\right]\left[1-\gamma v + \gamma\left(\frac{\gamma+1}{2}\right)v^2+\cdots\right]$$

$$= \bar{Y}\left\{1-\gamma v+\gamma\left(\frac{\gamma+1}{2}\right)v^2+\gamma u-\gamma^2uv+\gamma\left(\frac{\gamma-1}{2}\right)u^2+e_y-\gamma ve_y+\gamma ue_y+\ldots\right\}$$

Approximating to second order, we get

$$t_{drg} = \bar{Y}\left\{1+\gamma(u-v)+\left(\frac{\gamma^2+\gamma}{2}\right)v^2+\left(\frac{\gamma^2-\gamma}{2}\right)u^2-\gamma^2uv+e_y-\gamma e_y(v-u)\right\}$$

$$= \bar{Y}\{1+\gamma[(g_1e_{x_1}+g_2e_{x_2})-(g_2e_{x_1}+g_1e_{x_2})]+\left(\frac{\gamma^2+\gamma}{2}\right)(g_2e_{x_1}+g_1e_{x_2})^2+\left(\frac{\gamma^2-\gamma}{2}\right)(g_1e_{x_1}+g_2e_{x_2})^2-\gamma^2[(g_1^2+g_2^2)e_{x_1}e_{x_2}+$$

$$g1g2ex12+ex22+ey-\gamma eyg2ex1+g1ex2-g1ex1-g2ex2$$

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$$= \bar{Y}\{1+ \gamma(g_1-g_2)e_{x_1} + (g_2-g_1)\gamma e_{x_2} + \left(\frac{\gamma^2+\gamma}{2}\right)\left(g_2{}^2 e_{x_1}{}^2 + 2g_1g_2e_{x_1}e_{x_2} + g_1{}^2e_{x_2}{}^2\right) + \left(\frac{\gamma^2-\gamma}{2}\right)\left(g_1{}^2e_{x_1}{}^2 + 2g_1g_2e_{x_1}e_{x_2} +\right.$$

g22ex22−γ2(g12+g22)ex1ex2+g1g2ex12+ ex22+ ey−γey[(g2−g1)ex1+(g1−g2)ex2]}

$$= \bar{Y}\{1+\gamma((g_1-g_2)e_{x_1} + (g_2-g_1)\gamma e_{x_2} + \frac{\gamma^2 g_2{}^2 e_{x_1}{}^2}{2} + \frac{2g_1g_2\gamma^2 e_{x_1}e_{x_2}}{2} + \frac{\gamma^2 g_1{}^2 e_{x_2}{}^2}{2} + \frac{\gamma g_2{}^2 e_{x_1}{}^2}{2} + \frac{2\gamma g_1g_2e_{x_1}e_{x_2}}{2} + \frac{\gamma g_2{}^2 e_{x_2}{}^2}{2} -$$

$$\gamma^2\left[(g_1^2+g_2^2)e_{x_1}e_{x_2} + g_1g_2\left(e_{x_1}^2 + e_{x_2}^2\right)\right] + e_y - (g_2-g_1)\gamma e_y e_{x_1} - (g_1-g_2)\gamma e_y e_{x_2} + \frac{\gamma^2 g_1{}^2 e_{x_1}{}^2}{2} + \frac{2\gamma^2 g_1g_2e_{x_1}e_{x_2}}{2} + \frac{\gamma^2 g_2{}^2 e_{x_2}{}^2}{2} -$$

$$\frac{\gamma g_1{}^2 e_{x_1}{}^2}{2} - \frac{2\gamma g_1g_2e_{x_1}e_{x_2}}{2} - \frac{\gamma g_2{}^2 e_{x_2}{}^2}{2}$$

$$= \bar{Y}\{1+(g_1-g_2)\gamma e_{x_1} + (g_2-g_1)\gamma e_{x_2} + \left[-\gamma^2 g_1g_2 + \frac{\gamma^2 g_2^2}{2} + \frac{\gamma g_2^2}{2} + \frac{\gamma^2 g_1^2}{2} - \frac{\gamma g_1^2}{2}\right]e_{x_1}{}^2 + \left[-\gamma^2 g_1 g_2 + \frac{\gamma^2 g_1^2}{2} + \frac{\gamma g_2^2}{2} + \frac{\gamma^2 g_2^2}{2} -\right.$$

γg12 2ex22+[−γ2g12+g22+g1g2γ2+ γg1g2+γ2g1g2−γg1g2]ex1ex2+ ey−(g2−g1)γeyex1−(g1−g2)γeyex2}

$$= \bar{Y}\{1+(g_1-g_2)\gamma e_{x_1} + (g_2-g_1)\gamma e_{x_2} + \left[-\gamma^2 g_1g_2 + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right)\right]e_{x_1}{}^2 + \left[-\gamma^2 g_1g_2 + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) +\right.$$

g22γ2+γ2ex22+[2γ2g1g2−γ2(g12+g22)]ex1ex2+ey+(g1−g2)γeyex1−(g1−g2)γeyex2} (8)

$$t_{drg} - \bar{Y} = \bar{Y}\{e_y + (g_1-g_2)\gamma e_{x_1} - (g_1-g_2)\gamma e_{x_2} + \left[-\gamma^2 g_1g_2 + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right)\right]e_{x_1}{}^2 + \left[-\gamma^2 g_1g_2 +\right.$$

g12γ2−γ2+g22γ2+γ2ex22+[2γ2g1g2−γ2(g12+g22)]ex1ex2+g1−g2γeyex1−(g1−g2)γeyex2} (9)

$$B(t_{drg}) = E\left(t_{drg} - \bar{Y}\right) = \bar{Y}\left\{\left[g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) + g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) - \gamma^2 g_1g_2\right]\lambda' C_x^2 + \left[g_1{}^2\left(\frac{\gamma^2-\gamma}{2}\right) + g_2{}^2\left(\frac{\gamma^2+\gamma}{2}\right) - \gamma^2 g_1g_2\right]\lambda C_x^2 + \right.$$

$$\left.(2\gamma^2 g_1g_2 - \gamma^2(g_1^2+g_2^2))\lambda' C_x^2 + (g_1-g_2)\gamma\lambda'\rho C_y C_x - (g_1-g_2)\gamma\lambda\rho C_y C_x\right\} \quad (10)$$

The Mean Square error is given as:

$$MSE(t_{drg}) = E\left(t_{drg} - \bar{Y}\right)^2 = \bar{Y}^2 E\{e_y^2 + 2(g_1-g_2)\gamma e_y e_{x_1} - 2(g_1-g_2)\gamma e_y e_{x_2} - 2(g_1-g_2)^2\gamma^2 e_{x_1}e_{x_2} + (g_1 -$$

g22γ2ex12−g1−g22γ2ex22

$$= \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{1-a^*}{1+a^*}\right)\gamma\lambda' C_y\rho C_x - 2\left(\frac{1-a^*}{1+a^*}\right)\gamma\lambda\rho C_y C_x - 2\left(\frac{1-a^*}{1+a^*}\right)^2\gamma^2\lambda' C_x^2 + \left(\frac{1-a^*}{1+a^*}\right)^2\gamma^2\lambda' C_x^2 + \left(\frac{1-a^*}{1+a^*}\right)^2\gamma^2\lambda C_x^2\right\}$$

$$= \bar{Y}^2\left\{\lambda C_y^2 - 2\left(\frac{1-a^*}{1+a^*}\right)\gamma(\lambda-\lambda')\rho C_y C_x + \left(\frac{1-a^*}{1+a^*}\right)^2\gamma^2(\lambda-\lambda')C_x^2\right\}$$

$$= \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a^*-1}{1+a^*}\right)(\lambda-\lambda')\gamma\rho C_y C_x + \left(\frac{a^*-1}{1+a^*}\right)^2\gamma^2(\lambda-\lambda')C_x^2\right\} (11)$$

To obtain the optimal Mean Square Error, we differentiate (11) with respect to γ and set the resulting expression to zero. Thus,

$$\frac{\partial MSE(t_{drg})}{\partial\gamma} = 2\left(\frac{a^*-1}{1+a^*}\right)(\lambda-\lambda')\rho C_y C_x + 2\left(\frac{a^*-1}{1+a^*}\right)^2\gamma(\lambda-\lambda')C_x^2 = 0$$

$$\Rightarrow \rho C_y C_x + \left(\frac{a^*-1}{1+a^*}\right)\gamma C_x^2 = 0$$

$$\Rightarrow k + \left(\frac{a^*-1}{1+a^*}\right)\gamma = 0$$

$$\therefore \left(\frac{a^*-1}{1+a^*}\right)\gamma = -k$$

$$(g_2-g_1)\gamma = -k\gamma = \frac{k}{g_1-g_2} = k\frac{(1+a^*)}{(1-a^*)} (12)$$

Putting (12) into (11) gives the optimum MSE as:

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ISSN (online): 2581-3048

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https://doi.org/10.47001/IRJIET/2023.712027

$$\mathrm{MSE_{opt}}\,(t_{drg}) = \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a^*-1}{1+a^*}\right)\left(\frac{1+a^*}{1-a^*}\right)(\lambda-\lambda')k\rho C_y C_x + \left(\frac{a^*-1}{1+a^*}\right)^2\left(\frac{1+a^*}{1-a^*}\right)^2 k^2(\lambda-\lambda')C_x^2\right\}$$

$$= \bar{Y}^2\{\lambda C_y^2 + 2(\lambda-\lambda')\rho C_y C_x\,k + k^2(\lambda-\lambda')C_x^2\}$$

$$= \bar{Y}^2\{\lambda C_y^2 - 2(\lambda-\lambda')\rho C_y C_x\,k + k^2(\lambda-\lambda')C_x^2\} \qquad = \bar{Y}^2\left\{\lambda C_y^2 - 2(\lambda-\lambda')\rho^2 {C_y}^2 + (\lambda-\lambda')\rho^2 {C_y}^2\right\}$$

$$= \bar{Y}^2 C_y^2\,\{\lambda-(\lambda-\lambda')\rho^2\}(13)$$

Remark 1

(13), is similar to the variance of the regression estimator of population mean in double sampling. Therefore, Optimal Mean Square Error (OMSE) of the suggested generalized efficient family of ratio estimator of population mean has the same efficiency as the classical regression estimator in double sampling.

Table 2: Some members of the generalized efficient family of the ratio estimator (case I)

S/N	Estimators	a^*	ϒ
1	$t_{drg1} = \bar{y}$ sample Mean	0	0
2	$t_{drg2} = \bar{y}\,(\frac{\bar{x}_1}{\bar{x}_2})$, Sukhatme (1962)	0	1
3	$t_{drg3} = \bar{y}\,(\frac{\bar{x}_1}{\bar{x}_2})^2$	0	2
4	$t_{drg4} = \bar{y}\sqrt{\frac{\bar{x}_1}{\bar{x}_2}}$	0	$\frac{1}{2}$
5	$t_{drg5} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^k$	0	k
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1+(\frac{1-2k}{1+2k})\bar{x}_2}{(\frac{1-2k}{1+2k})\bar{x}_1+\bar{x}_2}\right)^{\frac{1}{2}}$	$\frac{1-2k}{1+2k}$	$\frac{1}{2}$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1+\frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1+\bar{x}_2}\right)^{3k}$	$\frac{1}{2}$	3k

Table 3: Some members of the generalized family of the ratio estimator with bias (case I)

S/N	Estimators	Bias
1.	$t_{drg1} = \bar{y}$, Sample Mean	Unbiased
2.	$t_{drg2} = \bar{y}\,(\frac{\bar{x}_1}{\bar{x}_2})$, Sukhatme (1962)	$\bar{Y}[-\lambda' C_x^2 + (\lambda'-\lambda)\rho C_y C_x]$
3.	$t_{drg3} = \bar{y}\,(\frac{\bar{x}_1}{\bar{x}_2})^2$	$\bar{Y}[(-3\lambda'+\lambda)C_x^2 + 2(\lambda'-\lambda)\rho C_y C_x]$
4	$t_{drg4} = \bar{y}\sqrt{(\frac{\bar{x}_1}{\bar{x}_2})}$	$\bar{Y}[-\frac{1}{8}(3\lambda'+\lambda)C_x^2 + \frac{1}{2}(\lambda'-\lambda)\rho C_y C_x]$
5	$t_{drg5} = \bar{y}\,(\frac{\bar{x}_1}{\bar{x}_2})^k$,	$\bar{Y}\{[(k-1)\lambda-(k+1)\lambda']\frac{k}{2}C_x^2 + (\lambda'-\lambda)k\rho C_y C_x\}$
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1+\left(\frac{1-2k}{1+2k}\right)\bar{x}_2}{\left(\frac{1-2k}{1+2k}\right)\bar{x}_1+\bar{x}_2}\right)^{\frac{1}{2}}$	$\bar{Y}\{\left[\frac{-k(2k-1)\lambda'}{8}+\frac{k(k-1)\lambda}{2}\right]C_x^2 + (\lambda'-\lambda)k\rho C_y C_x$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1+\frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1+\bar{x}_2}\right)^{3k}$	$\bar{Y}\{[(k-1)\lambda+(k-1)\lambda']\frac{k}{2}C_x^2 + (\lambda'-\lambda)k\rho C_y C_x\}$

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

Table 4: Some members of the generalized efficient family of the ratio estimator with MSEs (case I)

S/N	Estimators	MSEs
1.	$t_{drg1} = \bar{y}$, Sample Mean	$\bar{Y}^2 \lambda C_y^2$
2.	$t_{drg2} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$, Sukhatme (1962)	$\bar{Y}^2[\lambda C_y^2 + (\lambda - \lambda')(C_x^2 - 2\rho C_y C_x)]$
3.	$t_{drg3} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^2$	$\bar{Y}^2[\lambda C_y^2 + 4(\lambda - \lambda')(C_x^2 - \rho C_y C_x)]$
4	$t_{drg4} = \bar{y}\sqrt{\left(\frac{\bar{x}_1}{\bar{x}_2}\right)}$	$\bar{Y}^2[\lambda C_y^2 + (\lambda - \lambda')(\frac{C_x^2}{4} - \rho C_y C_x)]$
5	$t_{drg5} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^k$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda - \lambda')(kC_x^2 - 2\rho C_y C_x)]$
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1 + \left(\frac{1-2k}{1+2k}\right)\bar{x}_2}{\left(\frac{1-2k}{1+2k}\right)\bar{x}_1 + \bar{x}_2}\right)^{\frac{1}{2}}$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda - \lambda')(kC_x^2 - 2\rho C_y C_x)]$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{3k}$	$\bar{Y}^2[\lambda C_y^2 + k(\lambda - \lambda')(kC_x^2 - 2\rho C_y C_x)]$

Remark 2:

It should be noted that the optimal value of $'a'$ can also be obtained from (12) by making $'a'$ the subject of the formula. Thus

$$a^* = \frac{\gamma - k}{\gamma + k} \qquad (14)$$

Case II

Supposing that the second sample of size n_2 was drawn independently of the preliminary one, then the suggested estimator would still be same, but the bias and MSE in this case would differ from that of Case I. The bias and MSE in this case were derived by setting

$$E(e_{x_1} e_{x_2}) = E(e_y e_{x_2}) = 0 \qquad (15)$$

Thus for case II of the suggested generalized efficient family of ratio estimator, the bias and mean square error were obtained and presented as follows:

$$B_2(t_{drg}) = \bar{Y}\left\{\left[{g_2}^2\left(\frac{\gamma^2+\gamma}{2}\right) + {g_1}^2\left(\frac{\gamma^2-\gamma}{2}\right) - \gamma^2 g_1 g_2\right]\lambda' C_x^2 + \left[{g_1}^2\left(\frac{\gamma^2-\gamma}{2}\right) + {g_2}^2\left(\frac{\gamma^2+\gamma}{2}\right) - \gamma^2 g_1 g_2\right]\lambda C_x^2 - (g_1 - g_2)\gamma\lambda\rho C_y C_x\right\}$$

$$B_2(t\ \ ) =$$

$$\bar{Y}\left\{\left[\frac{a^2}{(1+a)^2}\left(\frac{\gamma^2+\gamma}{2}\right) + \frac{1}{(1+a)^2}\left(\frac{\gamma^2-\gamma}{2}\right) - \frac{a}{(1+a)^2}\gamma^2\right]\lambda' C_x^2 + \left[\frac{1}{(1+a)^2}\left(\frac{\gamma^2-\gamma}{2}\right) + \frac{a^2}{(1+a)^2}\left(\frac{\gamma^2+\gamma}{2}\right) - \frac{a}{(1+a)^2}\gamma^2\right]\lambda C_x^2 - \left(\frac{1-a}{1+a}\right)\gamma\lambda\rho C_y C_x\right\} \qquad (16)$$

And the mean square error as:

$MSE_2(t_{drg})$

$$= \bar{Y}^2\{\lambda C_y^2 - 2(g_1 - g_2)\lambda\gamma\rho C_y C_x + (g_1 - g_2)^2\gamma^2\lambda' C_x^2 + (g_1 - g_2)^2\gamma^2\lambda C_x^2\}$$

$$= \bar{Y}^2\{\lambda C_y^2 - 2(g_1 - g_2)\lambda\gamma\rho C_y C_x + (g_1 - g_2)^2\gamma^2(\lambda + \lambda')C_x^2\}$$

$$= \bar{Y}^2\left\{\lambda C_y^2 - 2\left(\frac{1-a}{1+a}\right)\lambda\gamma\rho C_y C_x + \left(\frac{1-a}{1+a}\right)^2\gamma^2(\lambda + \lambda')C_x^2\right\} \qquad (17)$$

To get the optimum MSE, (16) is differentiated partially and the emergent expression is set to zero. Thus:

$$\frac{\partial MSE_2(t_{drg})}{\partial\gamma} = -2(g_1 - g_2)\lambda\rho C_y C_x + 2\gamma(g_1 - g_2)^2(\lambda + \lambda')C_x^2 = 0$$

$$= -\lambda\rho C_y C_x + \gamma(g_1 - g_2)(\lambda + \lambda')C_x^2 = 0$$

$$= -\lambda k C_x^2 + \gamma(g_1 - g_2)(\lambda + \lambda')C_x^2 = 0$$

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

$$= -\lambda k + \gamma(g_1 - g_2)(\lambda + \lambda') = 0$$

$$\gamma = \frac{\lambda k}{(g_1 - g_2)(\lambda + \lambda')} = \frac{\lambda k}{\left(\frac{1-a^*}{1+a^*}\right)(\lambda + \lambda')} = \frac{\lambda k(1 + a^*)}{(1 - a^*)(\lambda + \lambda')} \quad (18)$$

Putting (17) into equation (16) gives;

$$\mathrm{MSE}_{2_{opt}}\left(t_{drg}\right) = \bar{Y}^2\left\{\lambda C_y^2 - \frac{2\lambda k(g_1 - g_2)}{(g_1 - g_2)(\lambda + \lambda')} \times \rho C_y C_x + \frac{(g_1 - g_2)^2(\lambda + \lambda')(\lambda k)^2}{(g_1 - g_2)^2(\lambda + \lambda')^2} C_x^2\right\}$$

$$= \bar{Y}^2\left\{\lambda C_y^2 - \frac{2\lambda^2\rho^2 C_y^2}{(\lambda + \lambda')} + \frac{\lambda^2\rho^2 C_y^2}{(\lambda + \lambda')}\right\}$$

$$= \bar{Y}^2\left\{\lambda C_y^2 - \frac{\lambda^2\rho^2 C_y^2}{(\lambda + \lambda')}\right\}$$

$$= \bar{Y}^2 C_y^2\left\{\lambda - \frac{\lambda^2}{(\lambda + \lambda')}\rho^2\right\}$$

$$= \bar{Y}^2 C_y^2\{\lambda - w\rho^2\} \qquad (19)$$

Where

$$w = \frac{\lambda^2}{(\lambda + \lambda')}$$

Few members of this estimator in case II are similar to those of case I, unless at optimal level where the values of $'a^{*'}$ and γ differs considerably. Table 5 shows some members of the estimator in Case II.

Table 5: Some members of the generalized efficient family of the ratio estimator (case II)

S/N	Estimators	a^*	γ
1	$\bar{y}$ sample Mean	0	0
2	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$, Sukhatme (1962)	0	1
3	$\bar{y}\sqrt{\frac{\bar{x}_1}{\bar{x}_2}}$	0	$\frac{1}{2}$
4	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^{\left(\lambda k/_{\lambda+\lambda'}\right)}$	0	$\frac{\lambda k}{\lambda + \lambda'}$
5	$\bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{3k}$	$\frac{1}{2}$	3k
6	$\bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^k$	0	k
7	$\bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{\frac{3\lambda k}{(\lambda+\lambda')}}$	$\frac{1}{2}$	$\frac{3\lambda k}{\lambda + \lambda'}$

Table 6: Some members of the generalized efficient family of the ratio estimator with bias (case II)

S/N	Estimators	Bias
1.	$t_{drg1} = \bar{y}$, Sample Mean	Unbiased
2.	$t_{drg2} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$, Sukhatme (1962)	$\bar{Y}(-\lambda\rho C_y C_x)$
3	$t_{drg3} = \bar{y}\sqrt{\left(\frac{\bar{x}_1}{\bar{x}_2}\right)}$,	$\bar{Y}[-\frac{1}{8}(3\lambda + \lambda')C_x^2 - \frac{1}{2}\lambda\rho C_y C_x]$
4	$t_{drg4} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^{\frac{\lambda k}{\lambda+\lambda'}}$,	$\bar{Y}\{[\frac{\lambda^2k^2 - \lambda k(\lambda+\lambda')}{2(\lambda+\lambda')}]C_x^2 - \frac{\lambda^2 k}{(\lambda+\lambda')}\rho C_y C_x\}$
5	$t_{drg5} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{3k}$	$\bar{Y}\{\frac{k}{2}[(k-1)(\lambda' + \lambda)]C_x^2 - \lambda k\rho C_y C_x\}$
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^k$	$\bar{Y}\{\left[\frac{k}{2}(k-1)\lambda' + \frac{k}{2}(k-1)\lambda\right]C_x^2 - \lambda k\rho C_y C_x\}$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1 + \frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1 + \bar{x}_2}\right)^{\frac{3\lambda k}{(\lambda+\lambda')}}$	$\bar{Y}\{\frac{[\lambda k(\lambda k - \lambda - \lambda')]}{2(\lambda+\lambda')}C_x^2 - \frac{\lambda^2 k\rho C_y C_x}{(\lambda+\lambda')}\}$

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

Table 7: Some members of the generalized efficient family of the ratio estimator with MSEs (case II)

S/N	Estimators	MSEs
1.	$t_{drg1} = \bar{y}$, Sample Mean	$\bar{Y}^2 \lambda C_y^2$
2.	$t_{drg2} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$,Sukhatme (1962)	$\bar{Y}^2[\lambda C_y^2 + (\lambda + \lambda')C_x^2 - 2\lambda\rho C_y C_x]$
3	$t_{drg3} = \bar{y}\sqrt{\left(\frac{\bar{x}_1}{\bar{x}_2}\right)}$	$\bar{Y}^2[\lambda C_y^2 + \frac{(\lambda+\lambda')}{4}C_x^2 - \lambda C_x]$
4	$t_{drg4} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^{\frac{\lambda k}{\lambda+\lambda'}}$,	$\bar{Y}^2[\lambda C_y^2 + \frac{\lambda^2 k^2(\lambda+\lambda')}{(\lambda+\lambda')^2}C_x^2 - 2\frac{\lambda^2 k}{(\lambda+\lambda')}\rho C_y C_x]$
5	$t_{drg5} = \bar{y}\left(\frac{\bar{x}_1+\frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1+\bar{x}_2}\right)^{3k}$	$\bar{Y}^2[\lambda C_y^2 + (\lambda + \lambda')k^2 C_x^2 - 2\lambda k\rho C_y C_x]$
6	$t_{drg6} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)^k$	$\bar{Y}^2[\lambda C_y^2 + (\lambda + \lambda')C_x^2 - 2\lambda k\rho C_y C_x]$
7	$t_{drg7} = \bar{y}\left(\frac{\bar{x}_1+\frac{1}{2}\bar{x}_2}{\frac{1}{2}\bar{x}_1+\bar{x}_2}\right)^{\frac{3\lambda k}{(\lambda+\lambda')}}$	$\bar{Y}^2[\lambda C_y^2 + \frac{\lambda^2 k^2(\lambda+\lambda')}{(\lambda+\lambda')^2}C_x^2 - 2\frac{\lambda^2 k}{(\lambda+\lambda')}\rho C_y C_x]$

2.4 Theoretical underpinnings

Bias, Relative Bias and Mean Square Errors of t_{drg}

If t_{drg} is the generalized efficient family of ratio estimator of the population mean $\bar{Y}$under Double Sampling Techniques, then the biases, relative biases, and MSEs is given as:

(a) Bias

$$B(t_{drg}) = [E(t_{drg}) - \bar{Y}] \qquad (20)$$

(b) Relative Biases

$$RB(t_{drg}) = \frac{[E(t_{drg}) - \bar{Y}]}{\bar{Y}}, \qquad (21)$$

(c) MSEs

$$MSE(t_{drg}) = E[(t_{drg}) - \bar{Y}]^2 \qquad (22)$$

(d) Relative Efficiency RE

Let $MSE(t_{drg})_{opt}$ be the MSE of the generalized efficient family of ratio estimator of the population mean $\bar{Y}$ and $MSE(\bar{y}_{dr})_{opt}$ be the MSE of the classical ratio estimator under optimal condition, then $MSE(t_{drg})_{opt}$ is said to be more efficient than $MSE(\bar{y}_{dr})_{opt}$, if;

(1) $\frac{MSE(t_{drg})_{opt}}{MSE(\bar{y}_{dr})_{opt}} < 1$ or $\frac{1}{\frac{MSE(t_{drg})_{opt}}{MSE(\bar{y}_{dr})_{opt}}} > 1$ (23)

(2) $MSE(\bar{y}_{dr})_{opt} - MSE(t_{drg})_{opt} > 0$ (24)

(e) Percentage Relative Efficiency PRE

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

$MSE(t_{drg})_{opt}$ is said to be more efficient than $MSE(\bar{y}_{dr})_{opt}$, in terms of PRE, if;

$$\frac{MSE(t_{drg})_{opt}}{MSE(\bar{y}_{dr})_{opt}} \times 100 < 100 \ or \ \frac{1}{\frac{MSE(t_{drg})_{opt}}{MSE(\bar{y}_{dr})_{opt}}} \times 100 > 100 \qquad (25)$$

Remarks 3:

(i) An estimator is said to be more efficient over its competitor's estimators if the estimator in question is the dominant estimator or one with the minor Mean Square Error.

(ii) An estimator is said to be Percentage Relative Efficient in relation to its brethren or existing estimators of same group if the estimator is the one with the smallest PRE or the largest PRE vice versa.

2.5 Comparison in terms of efficiency

The AOE is uniformly better than any other ratio estimator in double sampling, whose efficiency is not equal to or greater than the classical regression estimator. For other members of the suggested generalized efficient family, a member say t_{drgi} is better than t_{drgj} iff;

$$MSE(t_{drgi}) < MSE(t_{drgj}) \qquad (26)$$

$$\Rightarrow \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a_i-1}{a_i+1}\right)\gamma_i(\lambda-\lambda')\rho C_y C_x + \left(\frac{a_i-1}{a_i+1}\right)^2 \gamma_i^2(\lambda-\lambda')C_x^2\right\}$$

$$\leq \bar{Y}^2\left\{\lambda C_y^2 + 2\left(\frac{a_j-1}{a_j+1}\right)\gamma_j(\lambda-\lambda')\rho C_y C_x + \left(\frac{a_j-1}{a_j+1}\right)^2 \gamma_j^2(\lambda-\lambda')C_x^2\right\}$$

$$\Rightarrow 2\left[\left(\frac{a_i-1}{a_i+1}\right)\gamma_i - \left(\frac{a_j-1}{a_j+1}\right)\gamma_j\right] + \left[\left(\frac{a_i-1}{a_i+1}\right)^2\gamma_i^2 - \left(\frac{a_j-1}{a_j+1}\right)^2\gamma_j^2\right] < 0$$

$$\Rightarrow 2[m_i\gamma_i - m_j\gamma_j] + [m_i^2\gamma_i^2 - m_j^2\gamma_j^2] < 0$$

$$\Rightarrow 2[m_i\gamma_i - m_j\gamma_j] + [(m_i\gamma_i + m_j\gamma_j)(m_i\gamma_i - m_j\gamma_j)]$$

$$\Rightarrow (m_i\gamma_i - m_j\gamma_j)(2 + m_i\gamma_i + m_j\gamma_j) < 0 \ (27)$$

$$m_i = \frac{a_i-1}{a_i+1}, \quad m_j = \frac{a_j-1}{a_j+1}$$

(27) implies that either

$$m_i\gamma_i - m_j\gamma_j < 0 \ \ and \ 2 + m_i\gamma_i + m_j\gamma_j > 0$$

$$\text{or} \ \ m_i\gamma_i - m_j\gamma_j > 0 \ \ and \ 2 + m_i\gamma_i + m_j\gamma_j < 0$$

$$\Rightarrow \ m_i\gamma_i < m_j\gamma_j \ \text{ and } 2 > -(m_i\gamma_i + m_j\gamma_j) \text{ or}$$

$$m_i\gamma_i > m_j\gamma_j \text{ and } 2 < -(m_i\gamma_i + m_j\gamma_j) \qquad (28)$$

When (28) holds, then t_{drgi} is more efficient than t_{drgj}

3. Numerical Application

To establish the truthfulness or accuracy of the theoretical underpinnings of the study, Four (4) natural populations were employed from secondary sources. The populations alongside their sources and the MSE of some existing ratio estimators are as presented in table 8 and table 9 respectively.

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

Table 8: Populations and their parameters

	Population[Sources]	N	n_1	n_2	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ
I	[Cingi (2007)]	923	400	200	436.3	11440.5	1.72	1.86	0.955
II	[Murthy (1967)]	80	30	10	5182.64	1126.46	0.35426	0.75067	0.9413
III	[Kardilar & Cingi (2006)]	104	40	20	625.37	13.93	1.866	1.653	0.865
IV	[Handique (2012)]	2500	200	25	4.63	21.09	0.95	0.98	0.79

Table 9: Mean square errors of some existing ratio estimators in two phase sampling

Estimators	Populations			
	I	II	III	IV
$\bar{y}$, Sample mean	2,196.30	294,954.05	55,014.70	0.766
Sukhatme (1962)	934.83	407,572. 37	29,557.12	0.383
Singh & Vishwakarma (2007)	1153.96	98,876.96	35,607.08	0.395
Classical Regression	912.27	95,736.20	29,542.31	0.344

3.1 Empirical Results

Table 10: Biases and relative bias of some members of the generalized efficient family of ratio estimators (Case I)

Estimators	Populations			
	I	II	III	IV
t_{drg1}	0 (0)	0 (0)	0 (0)	0 (0)
t_{drg2}	-5.445(-0.0125)	-147.277(-0.028)	-68.028(-0.108)	-0.139(-0.0301)
t_{drg3}	-0.468(-0.00107)	73.269(0.0141)	-10.044(0.044)	0.0632(0.0136)
t_{drg4}	-3.194(-0.00732)	-97.987(-0.0189)	-39.354(-0.0629)	-0.00892(-0.019)
t_{drg5}	-5.0039(-0.1146)	-89.466(-0.0172)	-66.920(-0.107)	-0.1208(-0.0261)
t_{drg6}	-3.425(-0.0078)	-69.605(-0.0134)	-44.586(-0.0713)	-0.1081(-0.0233)
t_{drg7}	1.446(0.0033)	-12.068(-0.0023)	24.537(0.0392)	-0.0216(-0.0047)

The relative bias are enclosed in braces.

Table 11: MSE of some members of the generalized efficient family of estimators (Case I)

Estimators	Populations			
	I	II	III	IV
t_{drg1}	2,196.30	294,954.1	55,014.70	0.766
t_{drg2}	934.83	407,572.37	29,557.12	0.383
t_{drg3}	2,966.04	2,539,283.15	57,530.10	1.441
t_{drg4}	1,153.93	98,876.96	35,607.08	0.395
t_{drg5}	912.27	95,736.20	29,542.31	0.344
t_{drg6}	912.27	95,736.20	29,542.31	0.344
t_{drg7}	912.27	95,736.20	29,542.31	0.344

Table 12: Bias and relative bias of some members of the generalized efficient family of ratio estimator (Case II)

Estimators	Populations			
	I	II	III	IV
t_{drg1}	0 (0)	0 (0)	0 (0)	0 (0)
t_{drg2}	-5.198(-0.0119)	-113.516(-0.021)	-67.409(-0.1079)	-0.135(-0.029)
t_{drg3}	-5.0719(-0.0116)	-160.178(-0.031)	-62.881(-0.1005)	-0.1360(-0.0291)
t_{drg4}	-3.378(-0.0077)	-41.166(-0.0079)	-47.689(-0.0762)	-0.0925(-0.0199)
t_{drg5}	-5.0039(-0.0115)	-89.467(-0.0172)	-66.919(-0.1070)	-0.1208(-0.0261)
t_{drg6}	-0.4234(-0.0009)	-39.0528(-0.007)	-1.199(-0.0019)	-0.0399(-0.0086)
t_{drg7}	3.378(0.0077)	40.312(0.0077)	47.627(0.0761)	0.0924(0.0199)

International Research Journal of Innovations in Engineering and Technology (IRJIET)
ISSN (online): 2581-3048
Volume 7, Issue 12, pp 189-202, December-2023
https://doi.org/10.47001/IRJIET/2023.712027

Table 13: MSE of some members of the generalized efficient family of ratio estimators (Case II)

Estimators	Populations			
	I	II	III	IV
t_{drg1}	2,196.30	294,954,05	55,014.70	0.766
t_{drg2}	1,150.29	757,513.14	30,331.86	0.427
t_{drg3}	800.703	116,436.69	27,766.15	0.369
t_{drg4}	722.33	83,804.54	25,211.82	0.337
t_{drg5}	912.27	95.729.74	29.543.56	0.343
t_{drg6}	912.27	95,729.74	29,543.56	0.343
t_{drg7}	722.33	83,804.54	25,211.82	0.337

Table 14: *RE* and *PRE* of some members of the generalized efficient family of ratio estimators (Case I)

Estimators	Populations			
	I	II	III	IV
$\bar{y}$ Sample mean t_{drg1}	1(100)	1(100)	1(100)	1(100)
Sukhatme (1962)	0.5254(53)	0.3352(33)	0.6472(65)	0.5156(52)
Singh & Vishwakarma (2007)	0.4256(43)	1.3818(138)	0.5272(53)	0.5000(50)
Classical Regression	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg2}	0.5273(53)	2.5682(256)	0.5513(55)	0.5574(56)
t_{drg3}	0.3645(36)	0.3947(39)	0.5047(50)	0.4817(48)
t_{drg4}^{*}	0.3288(33)	0.2841(28)	0.4582(46)	0.4399(44)
t_{drg5}	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg6}	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg7}	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)

PRE are the values enclosed in braces

Table 15: *RE* and *PRE* of some members of the generalized efficient family of ratio estimators (Case II)

Estimators	Populations			
	I	II	III	IV
$\bar{y}$ Sample mean t_{drg1}	1(100)	1(100)	1(100)	1(100)
Sukhatme (1962)	0.5254(53)	0.3352(33)	0.6472(65)	0.5156(52)
Singh & Vishwakarma (2007)	0.4256(43)	1.3818(138)	0.5272(53)	0.5000(50)
Classical Regression	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg2}	0.5273(53)	2.5682(256)	0.5513(55)	0.5574(56)
t_{drg3}	0.3645(36)	0.3947(39)	0.5047(50)	0.4817(48)
t_{drg4}^{*}	0.3288(33)	0.2841(28)	0.4582(46)	0.4399(44)
t_{drg5}	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg6}	0.4153(42)	0.3245(32)	0.5370(54)	0.4477(44)
t_{drg7}	0.3288(33)	0.2841(28)	0.4582(46)	0.4477(45)

PRE are the values enclosed in braces

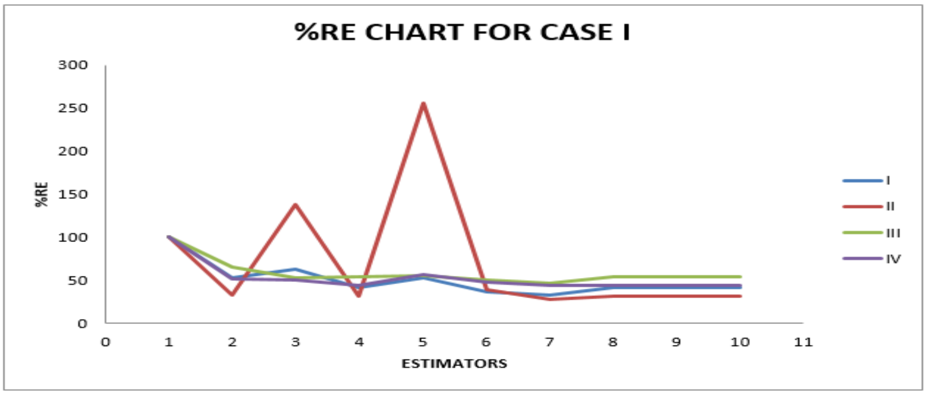

Figure 1: PRE of the estimators Case I

International Research Journal of Innovations in Engineering and Technology (IRJIET)

ISSN (online): 2581-3048

Volume 7, Issue 12, pp 189-202, December-2023

https://doi.org/10.47001/IRJIET/2023.712027

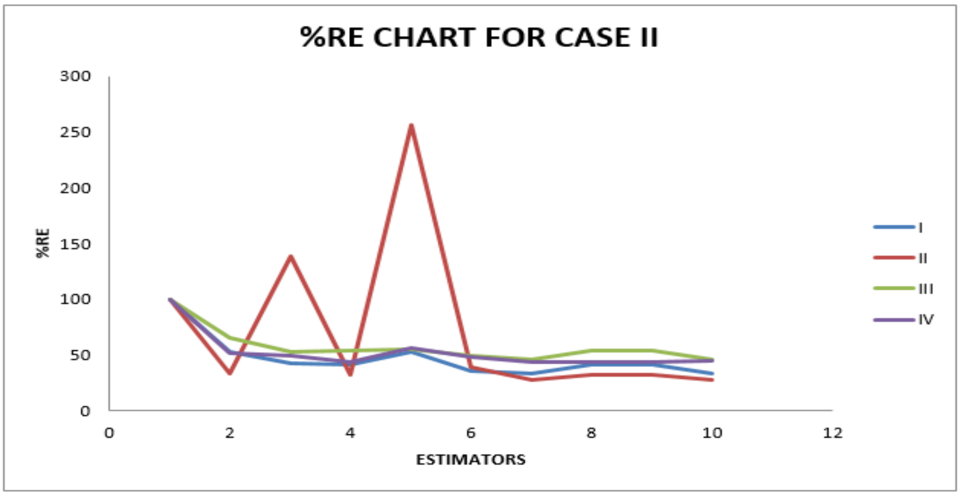

Figure 2: PRE of the estimators Case II

3.2 Discussion of Result

In this study, a generalized efficient family of ratio estimator for population mean based on a single supplementary variable, under double sampling techniques was suggested and presented in (5). Some members of the suggested estimator were obtained by varying the scalars $a^{*}$and γ which helped in designing the estimator. The properties of members of the estimator such as bias and MSE were then derived and presented alongside its members in tables 2, 3 and 4, for case I and tables 5, 6, and 7 for case II of the double sampling techniques, from where it was found that the suggested estimator produced some of the existing ratio estimators such as the sample mean $\bar{y}$, the Sukhatme (1962) classical ratio estimator, Singh & Vishwakarma (2007), which were presented in table 1.

The biases and MSEs of the members of the generalized efficient family of ratio estimator were inferred from the derived bias and MSE of the mother or generator estimator which were shown in (10), (16) for biases and (11), (17) for MSEs, case I and case II respectively. Optimality condition was established and Optimal Mean Square Errors were derived for both cases 1 and II and presented in (13) and (19) respectively.

Theoretical underpinnings bordering on efficiency, Relative Efficiency (RE) and Percentage Relative Efficiency (PRE)of estimator and conditions for which the estimator would be uniformly better than its competitor's estimators were formulated and shown in (23), (24), and (25) correspondingly. In order to establish the truthfulness or accuracy of the theoretical underpinnings of the study, Four (4) populations' data sets were employed from secondary sources. The populations alongside their sources and the MSEs of some existing ratio estimators is presented in table 8 and 9 respectively.

The biases and MSEs were then computed and presented in table 10, 11 for case I and table 12, 13 for case II. It was observed from table 11 that MSEs of the members of the estimators$t_{drg\,1}$,$t_{drg\,2}$, t_{drg3} and $t_{drg\,5}$in case I were in tandem with the simple random sample mean, Sukhatme (1962), Singh and Vishwarkama (2007), and the classical regression estimator in double sampling. However estimators $t_{drg\,4}$,$t_{drg\,7}$ in case II performed uniformly better than $t_{drg\,1}$,$t_{drg\,2}$, $t_{drg\,3}t_{drg\,5}$ and the classical regression estimator in double sampling and were therefore said to be dominant estimators or ones with the minor Mean Square Errors.

$t_{drg\,4}$,$t_{drg\,7}$ in case II also produced smaller PREs in relation to its brethren or estimators of same group. This generalization can be envisaged in table 14, table 15 and Figures 1, 2, for cases 1, II respectively.

4. Conclusion

A generalized efficient family of ratio estimator for population mean based on a single supplementary variable, under double sampling techniques was suggested, theoretical underpinnings bordering efficiency of the estimator in question was formulated and validated in entirety. The estimator showed significant gain in efficiency and superiority over its brethren at optimal condition. This gain in efficiency is more significant when the second sample of size n_2 was drawn independently of the preliminary one. Therefore, sub-sampling independent of the first phase sample is recommended for higher efficiency.

ISSN (online): 2581-3048
Volume 7, Issue 12, pp 189-202, December-2023
https://doi.org/10.47001/IRJIET/2023.712027

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ISSN (online): 2581-3048
Volume 7, Issue 12, pp 189-202, December-2023
https://doi.org/10.47001/IRJIET/2023.712027

Citation of this Article:

Etorti, Imoke John, Agbebia, Catherine Shike, Richmond Ofonodo, Edet Effiong Bassey, "A Generalized Efficient Family of Ratio Estimator of Population Mean Under Double Sampling Techniques" Published in *International Research Journal of Innovations in Engineering and Technology - IRJIET*, Volume 7, Issue 12, pp 189-202, December 2023. Article DOI https://doi.org/10.47001/IRJIET/2023.712027

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