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Use the Method of Spheres Inscribed in Cones to Solve Geometric Problems

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***Abstract* - In geometry, there are problems that if solved by conventional methods are very long and sometimes impossible to solve. But if you find a suitable configuration, solving problems will be much simpler. The following is a method to find a sphere inscribed in a cone to solve some advanced problems in geometry. This method allows combining the distance problem with the angle problem into an easily solvable tangent problem.**

Keywords: Sphere: sp, Cone: co, Plane: pl.

I. INTRODUCTION

- If a pl is tangent to the co, it is also tangent to the sp. This common tangent pl will contain a generating line of the co. (see Fig.1. a)
- If a pl passes through the apex of the co and is tangent to the sp, then it is also tangent to the co. (see Fig.1. a)
- There are many spheres inscribed in a cone. The centers of these spheres lie on the axis of the cone. (See Fig.1.b). the Hyos in shaping devices and structures in the world.

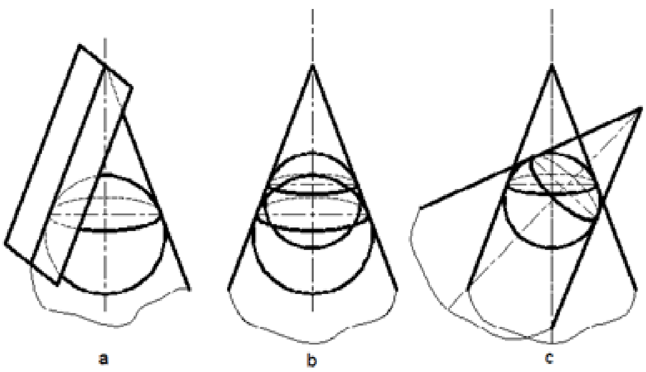

Figure 1

II. APPPLICATION TO SOLVE SOME GEOMETRIC PROBLEMS

2.1 Problem 1

Draw end the end generators of a circular right co with SH as the axis of revolution and the vertex angle as 60 degrees. (see Fig.2.)

We will replace the circle at the base of the co with a sphere. The sp has its center on the axis of the circular co and is in contact with the cone surface along the co's base circle.

Step 1: Determine the true length of SH

Step 2: Construct the right triangle SHM with angle MHS equal to 30 degrees. HM is the radius of the sph with center H in contact with the co.

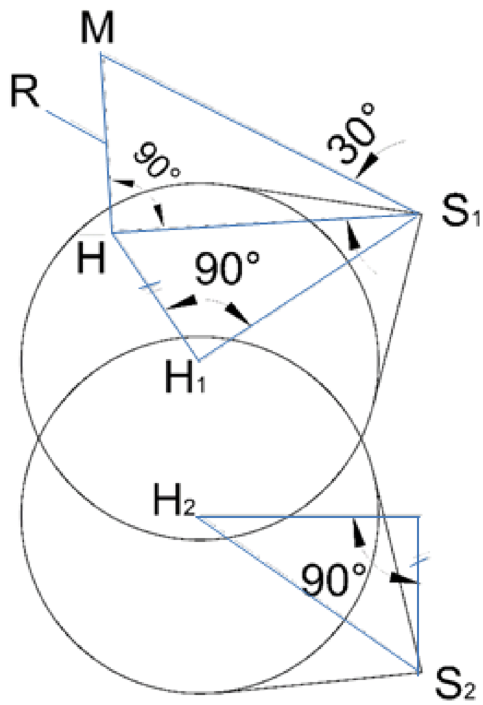

Figure 2

2.2 Problem 2

Given line a and pl Q that does not pass through a. (see Fig.3.)

Construct pl P that satisfies the two constraints listed below:

The distance from P to a equals to r

The angle between P and Q equals to 600

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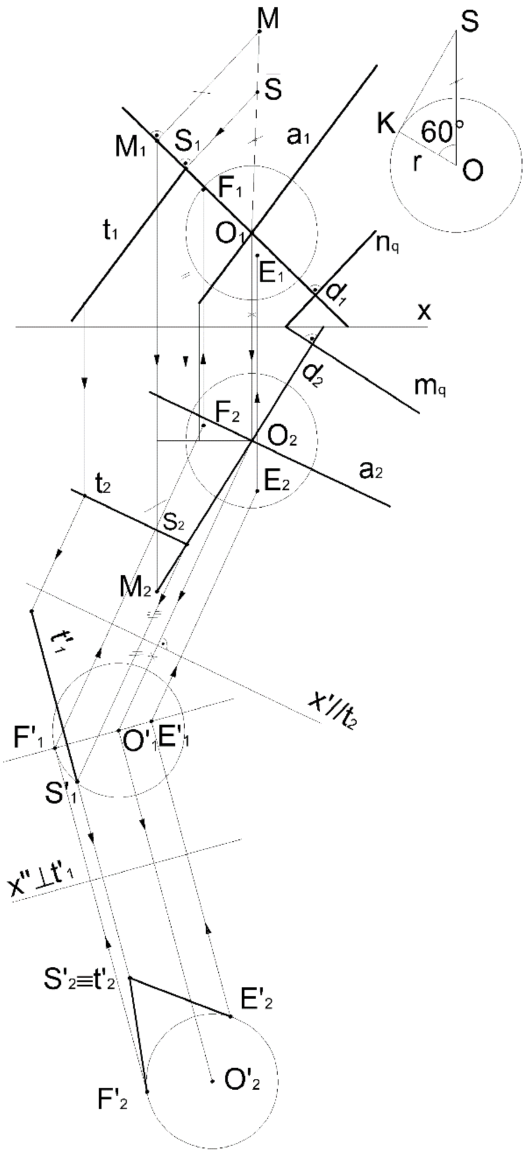

Figure 3

Step 1: Determine the true length of SH

Step 2: Construct the right triangle SHM with angle MHS equal to 30 degrees. HM is the radius of the sph with center H in contact with the co.

The solution:

- Take point O on line a.
- Draw line d through point O and perpendicular to plane Q.
- Construct a sphere with center O and radius r.
- Construct a right circular cone whose axis is line d and tangent to the sphere
- Draw line t passing through vertex S of the cone and parallel to line a.
- Construct two planes passing through t and tangent to the sphere.
- The problem has two solutions: two planes (t,E) and (t,F) (See Fig.3.)

2.3 Problem 3

Two skew lines a and b and a point M were given.

Construct pl P passing through M and tangent to two right circular cylinders with axes a and b. Determine the radius of those two cylinders.

The answer:

- Take point N on line a.
- Construct line c through point N and parallel to line b
- We will get the pl Q determined by two intersecting lines a and c
- Construct the horizontal line h of pl Q
- We use the method of replacing the projection pl so that the Q pl becomes the projecting pl.
- The results is that the new projections of two lines a and b are two parallel lines.
- P becomes a projecting plane. The new projections of it are two lines tangent to two circles with radius r and R.
- The radius R of the first right circular cylinder whose axis is line a equals the distance between two parallel lines a and P
- The radius r of the second one whose axis is line b equals the distance between two parallel lines b and P.
- The problem has two solutions

(See Figure 4 and Figure 5)

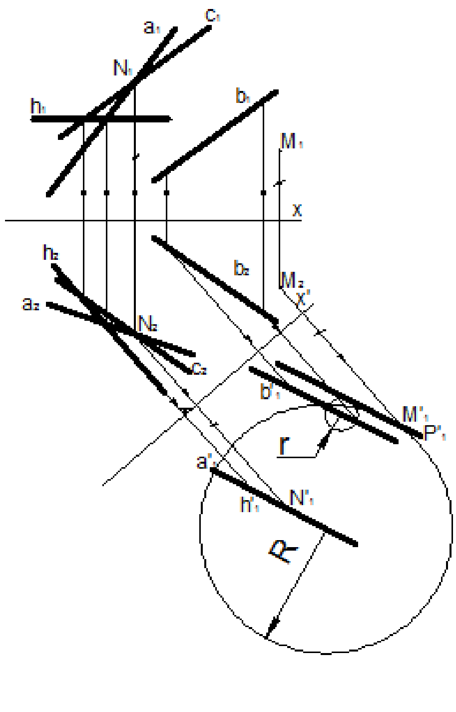

Figure 4

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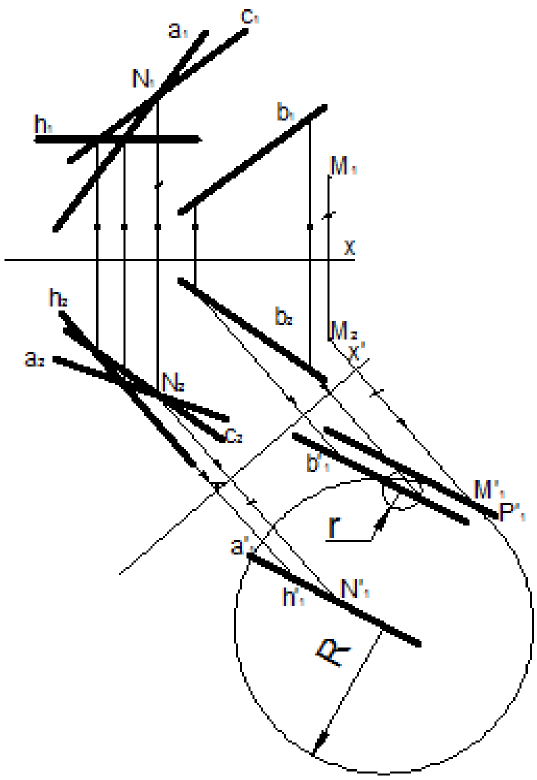

Figure 5

2.4 Problem 4

Given any 3 points A, B, C. Construct a straight line d through point A and the distance from C to d is 2/3 of the distance from D to d. (See Fig. 6)

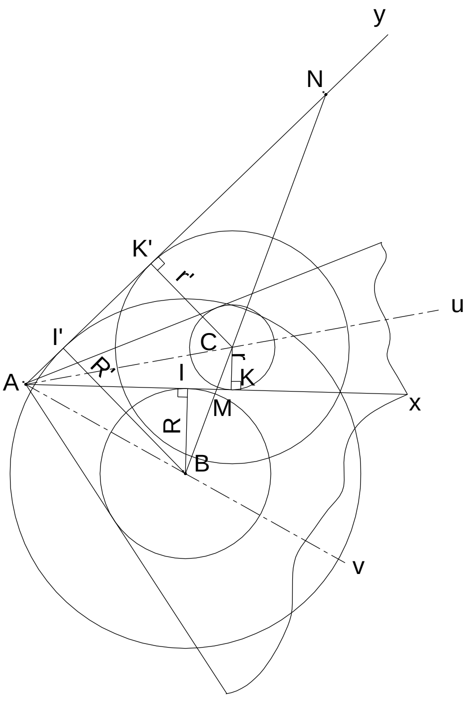

Figure 6

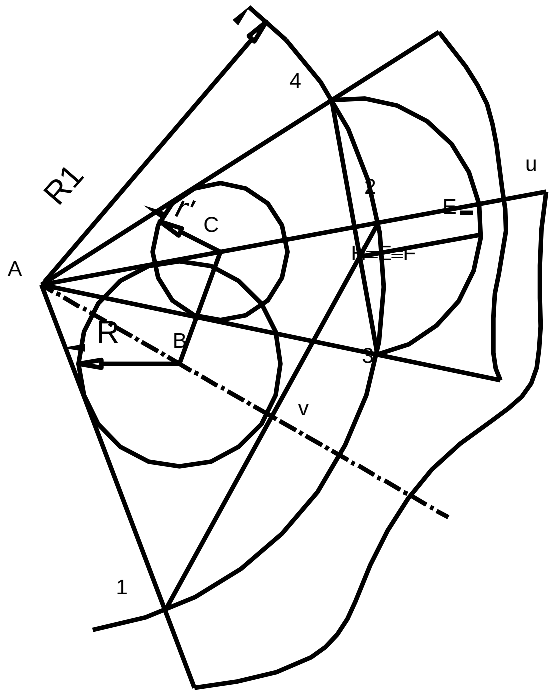

Figure 7

The answer:

Case 1:

- Divide segment BC into 3 equal parts.
- Take point M located in segment BC so that MC=2MB. Connecting A with M we get the straight line Ax.

Construct BI and CK respectively perpendicular to Ax.

In turn, construct a sp with center B and radius BI and a sp with center C and radius CK. BI=R, CK=r

The two co with vertex A are circumscribed by the two above sp, touching each other along the generating line Ax.

Ax is the straight line d to construct. The problem has a geometric solution.

- Similarly, take point N outside BC so that NC=2BC. Connecting A with N we get the straight line Ay.

Construct BI’ and CK’ respectively perpendicular to Ay.

In turn, construct a sp with center B and its radius equals BI’ and a sp with center C and radius of it equals CK’. BI’=R’, CK’=r’

The two co with vertex A are circumscribed by the two above sp, touching each other along the generating line Ay.

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Ay is the straight line d to construct. The problem has a geometric solution.

Case 2:

Construct a sp with center B and radius R'' (R<R''<R').

Construct a sp with center C and radius r'' such that r''=2/3R''. (r<r''<r').

Consider two co with vertex A circumscribing the above two sp.

Construct a sp with center A and radius R1.

This sp intersects 2 co in 2 circles 1-2 and 3-4 respectively. These two circles intersect at two points E and F.E and F are symmetrical about the pl(ABC). EF intersects pl (ABC) at H. Point H is the middle point of the straight line EF □ HE=HF.

The problem has 2 solutions: two straight lines AE and AF.

To find two points E and F, we revolve one circle above its diameter until it coincides with the pl (ABC).

If R'>BC+r''⇒ R'>$\frac{2}{3}R'' + BC$⇒R''>3BC, two co above do not intersect, so the problem has no solution.

If R''<R, two cones above do not intersect, so the problem has no solution. (See Figure 6 and Figure 7)

III. CONCLUSION

Geometry is a science that is widely applied in engineering and life. It is very complicated and there are many ways to solve it. To solve geometric problems, we must think for ourselves and find solutions. So writing articles about this subject is very difficult. Because it has no experiments, there are no cited experimental data. This article presents a new solution to solve some geometric problems. It helps us solve faster and easier to understand.

REFERENCES

[1] Nguyen Dinh Tri (editor-in-chief), Advanced Mathematics, volume 1, Education Publishing House 2006. ... Nguyen Thuy Thanh, Background of complex variable theory, VNU Publishing House, 2006.

[2] Dinh Tri Nguyen (editor) - Van Dinh Ta - Ho Quynh Nguyen ... Advanced Math Exercises Episode 1 - Dinh Tri Nguyen June 3, 2018In "Curriculum".

[3] Mechanics Program (Episode 2: Dynamics) – Sanh Do (Editor). September 2016 - publisher of Hanoi University of Science and Technology.

[4] Hoang Long, Banh Tien Long –Graphics 1, Publishing house of Ha Noi University of Science and Technology, 2016.

[5] Hoang Long, Banh Tien Long –Graphics 2, Publishing house of Ha Noi University of Science and Technology, 2016.

[6] Hoang Van Than (editor-n-chief), Doan Nhu Kim, Duong Tien Tho - Descriptive geometry, Science and Technology Publishing House, 2005.

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