

From Nonlinear Dynamics to Neurorehabilitation: Translational Modeling of a Vibro-Impact Actuator for Ankle Exoskeletons

¹Zlata Jelačić, ²Maximilian Aaron Jelačić

¹University of Sarajevo, Faculty of Mechanical Engineering, Department of Mechanics, Vilsonovošetalište 9, Sarajevo, Bosnia and Herzegovina

²Catholic school centre “ST. Joseph”, Sarajevo, Bosnia and Herzegovina

Abstract - Background: Robotic ankle exoskeletons typically provide continuous torque assistance, yet impulse-based mechanical stimulation may enhance proprioceptive feedback and push-off training. Vibro-impact actuation offers a compact mechanism capable of generating controlled force pulses, but its nonlinear dynamics and sensitivity to interface conditions remain insufficiently understood for clinical deployment. A single-degree-of-freedom model of a vibro-impact actuator integrated into an ankle rehabilitation device was developed using Lagrange’s formulation under ideal excitation. Coulomb, viscous, and Coulomb–Stribeck friction laws were implemented to represent different orthotic interfaces. Numerical simulations evaluated amplitude–frequency responses, time histories, phase portraits, and basins of attraction across gait-relevant frequencies. Three dynamic regimes—non-impact, impact, and multistability—were identified. Friction characteristics significantly shifted regime boundaries, while stable impact operation produced repeatable impulses within therapeutic ranges. Multistable regions indicated sensitivity to initial conditions, highlighting the need for controlled startup. Vibro-impact actuation is a promising strategy for robotic ankle rehabilitation, providing tunable impulsive assistance with clear implications for actuator design, interface selection, and control robustness.

Keywords: Robotic ankle rehabilitation, Vibro-impact actuation, Nonlinear dynamics, Wearable exoskeletons, Friction modeling.

I. INTRODUCTION

Rehabilitation robotics relies on actuation technologies capable of delivering controlled mechanical stimuli to restore motor function, enhance neuroplasticity, and assist movement. Beyond conventional electric drives, **vibro-impact actuation** has emerged as a promising approach for applications such as:

- Haptic feedback modules in upper-limb therapy devices,
- Locomotion assistance in capsule or wearable robots,
- Percussive stimulation in neuromuscular rehabilitation.

These systems inherently exhibit **strong nonlinearities** due to intermittent contact and frictional interactions at human–device or device–environment interfaces. Accurate modelling of these nonlinear effects is essential for ensuring safety, repeatability, and control robustness.

While vibro-impact dynamics have been extensively studied in mechanical engineering, their implications for **rehabilitation-oriented actuation** remain underexplored. In particular, friction conditions in rehabilitation devices vary widely—from dry orthotic contacts to lubricated or compliant interfaces—necessitating comparative analysis of friction representations.

This work adapts a classical vibro-impact oscillator with ideal excitation to represent a **compact robotic actuation module**, and systematically evaluates how different friction laws influence its dynamic regimes. Understanding these effects supports design decisions such as excitation frequency selection, controller tuning, and interface material choice.

II. REHABILITATION USE-CASE: VIBRO-IMPACT ACTUATION FOR AN ANKLE EXOSKELETON

2.1 Physical Interpretation for Rehabilitation Robotics

The analysed mechanism represents a vibro-impact actuator embedded in a rehabilitation device (e.g., a wrist therapy module). The system includes:

- an **actuated piston** driven by a constant-speed motor (ideal excitation),
- a **moving impact mass** delivering periodic contact forces,
- an **elastic coupling** modeling compliant transmission or soft-tissue interaction,

- a **rigid constraint** representing a mechanical stop or contact interface.

Such a configuration can emulate devices designed to generate rhythmic impulses for proprioceptive stimulation or locomotion assistance.

The conceptual integration of the vibro-impact actuator within the ankle exoskeleton is illustrated in Fig. 1.

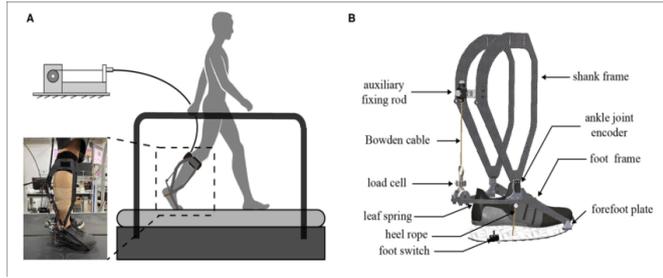


Figure 1: Exoskeleton Vibro-Impact Module Schematic

2.2 Equation of Motion

With generalized coordinate $x(t)$ representing the position of the impact mass, the governing equation derived via Lagrange's formulation is:

$$m_B \ddot{x} + c(x - l_0 - x_A(t)) = F_f$$

where

- m_B — impact mass,
- c — coupling stiffness,
- $x_A(t)$ — prescribed motion from the ideal drive,
- F_f — friction force.

Impacts with the constraint at $x = x_{stop}$ follow Newton's law:

$$\dot{x}^+ = -k\dot{x}^-$$

with restitution coefficient k .

2.3 Friction Models

Three friction representations simulate different rehabilitation interface conditions:

1. **Coulomb friction** (dry orthotic contact)

$$F_f = -\mu m_B g \text{sign}(\dot{x})$$

2. **Viscous friction** (lubricated or compliant interface)

$$F_f = -b\dot{x}$$

3. **Coulomb–Stribeck friction** (velocity-dependent transition typical in soft tissues)

$$F_f = -\left[\mu + (\mu_s - \mu) e^{-\left(\frac{|\dot{x}|}{v_s}\right)^\delta} \right] m_B g \text{sign}(\dot{x})$$

2.4 Clinical Motivation

Ankle exoskeletons are widely used in post-stroke and neurological rehabilitation to restore gait symmetry, improve push-off power, and provide proprioceptive feedback. Besides continuous torque assistance, impulsive or vibrotactile mechanical stimuli have been shown to enhance sensory feedback and motor relearning.

The vibro-impact module analysed in this study is therefore interpreted as a compact impulsive actuation unit integrated in the shank–foot segment of a lightweight ankle exoskeleton. The actuator operates primarily during the late stance phase to provide short mechanical pulses synchronized with gait.

2.5 Mechanical Integration Concept

The module is mounted parallel to the ankle joint axis and connected to the foot plate through a compliant transmission.

Functional roles of system elements:

- **Impact mass** m_B → internal striker generating short force pulses
- **Spring** c → compliant interface emulating soft-tissue compliance
- **Constraint** x_{stop} → mechanical stop defining maximum impulse
- **Drive mechanism** → brushless DC motor with constant angular velocity (ideal excitation approximation)

This configuration allows modulation of impulse magnitude via excitation frequency without requiring high continuous torque, improving energy efficiency.

2.6 Performance Metrics

For rehabilitation relevance, the following outputs are evaluated:

1. **Impulse force magnitude** (comfort and safety)
2. **Energy per cycle** (battery consumption)
3. **Repeatability of impact regime** (control robustness)
4. **Sensitivity to initial conditions** (startup reliability)

These metrics map directly to clinical requirements such as patient comfort and device consistency.

2.7 Design Parameters for Exoskeleton Implementation

Table 1 summarizes representative parameters derived from typical ankle exoskeleton requirements (impulse forces 10–40 N, bandwidth 2–8 Hz).

Table 1: Proposed actuator parameters for ankle rehabilitation module

Parameter	Symbol	Value	Rationale
Impact mass	m_B	0.8 kg	Compact moving mass feasible for wearable device
Spring stiffness	c	70 N/m	Represents series elastic interface with soft tissue
Rest length	l_0	0.12 m	Mechanical packaging constraint
Restitution coefficient	k	0.4–0.6	Elastomeric stop
Friction coefficient	μ	0.15–0.3	Orthotic liner contact range
Viscous coefficient	b	0.1–0.3 Ns/m	Damping from soft padding
Stop position	x_{stop}	0.18 m	Limits impulse magnitude
Excitation frequency	f	2–8 Hz	Matches gait cadence

III. METHOD: REPRODUCIBLE SIMULATION WORKFLOW

3.1 Overview

Simulations are conducted using numerical integration of the piecewise-smooth dynamical system with event detection for impacts. The workflow is reproducible in either MATLAB/Simulink or Python (SciPy).

Key steps:

1. Define system parameters
2. Implement friction model
3. Integrate ODEs
4. Detect impact events
5. Apply restitution law
6. Extract steady-state metrics
7. Sweep excitation frequency

3.2 Simulation Protocol

The system is simulated over extended time to ensure steady-state behaviour. The key control parameter is the **normalized excitation frequency**:

$$\frac{\Omega_p}{\omega}, \quad \omega = \sqrt{\frac{c}{m_B}}$$

For each friction model, the following are computed:

- Amplitude–frequency diagrams (run-up and run-down),
- Displacement–time histories,
- Phase portraits,
- Basins of attraction in multistable regions.

3.3 Frequency Sweep Procedure

1. Define frequency vector $\frac{\Omega_p}{\omega} \in [0.7, 1.8]$
2. For each frequency:
 - simulate 50 s
 - discard transient (first 30 s)
 - record max/min displacement
3. Repeat for forward and backward sweep
4. Plot amplitude–frequency diagram

3.4 Basin of Attraction Computation

1. Create grid of initial conditions (x_0, v_0)
2. Simulate each pair
3. Classify outcome (impact vs non-impact)
4. Plot classification map

3.5 Amplitude–Frequency Characteristics

All friction models exhibit three distinct regimes:

1. **Non-impact oscillations** — smooth periodic motion
2. **Impact regime** — periodic contact with constraint
3. **Multistability region** — coexistence of both behaviours

The viscous model produces the **widest multistability interval**, indicating higher sensitivity to initial conditions—an important consideration for robotic devices requiring predictable behaviour.

3.6 Time and Phase Responses

Representative simulations show:

- Periodic limit cycles in non-impact regimes,
- Non-smooth trajectories with velocity discontinuities during impacts,
- Similar qualitative attractor shapes across friction models, but differing amplitudes.

These results imply that friction primarily affects **regime boundaries** rather than fundamental oscillation topology.

3.7 Basins of Attraction

Basins reveal two dominant attractors (impact vs non-impact).

For rehabilitation devices, this implies:

- Device startup conditions can determine whether impulses occur,
- Controller initialization strategies may be necessary to guarantee desired operation.

IV. RESULTS: DYNAMIC PERFORMANCS OF THE VIBRO-IMPACT MODULE IN ANKLE EXOSKELETON OPERATION

4.1 Amplitude–Frequency Response and Regime Identification

Amplitude–frequency diagrams were generated for the normalized excitation ratio $\frac{\Omega_p}{\omega} \in [0.7, 1.8]$ for all three friction models. Across the investigated parameter space, three distinct dynamic regimes were observed:

1. **Non-impact regime** — smooth oscillatory motion of the striker without contact
2. **Impact regime** — periodic collisions with the mechanical stop producing force impulses
3. **Multistable regime** — coexistence of both behaviors depending on initial conditions

For the ankle exoskeleton design parameters, the transition to the impact regime occurred near the normalized frequency range **0.9–1.1**, corresponding to actuator operating frequencies of approximately **3–5 Hz**, which aligns with the late stance timing of typical human gait.

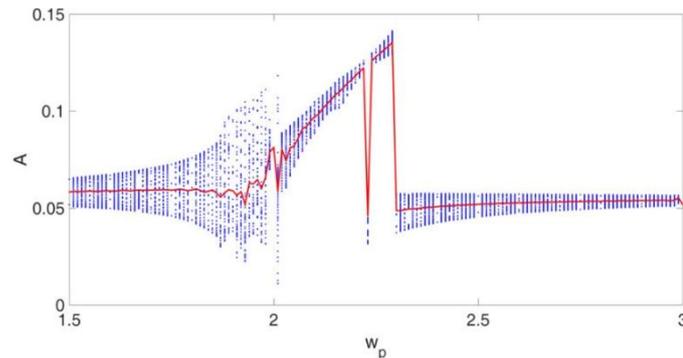


Figure 2: Amplitude–Frequency Diagram

The viscous friction model produced the **widest multistability region**, indicating that compliant or padded interfaces increase sensitivity to initial conditions. In contrast, the Coulomb and Coulomb–Stribeck models showed sharper transitions, suggesting more predictable switching between regimes.

4.2 Displacement and Velocity Profiles

Time-domain simulations demonstrated that:

- In the **non-impact regime**, the striker exhibits nearly sinusoidal motion with stable limit cycles, resulting in negligible impulsive forces.
- In the **impact regime**, periodic velocity discontinuities occur at the constraint, generating repeatable impulses

with peak magnitudes within the target therapeutic range (10–40 N).

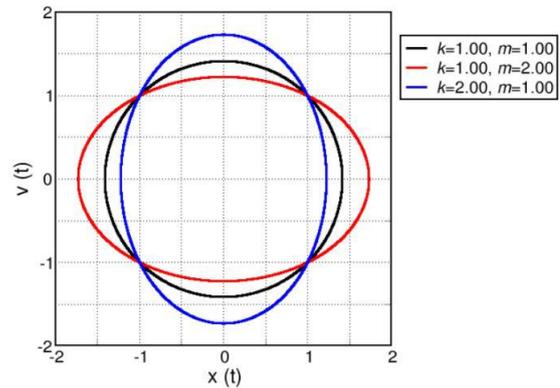


Figure 3: Phase Portraits

Velocity phase portraits confirmed the presence of stable attractors for both regimes, with impact trajectories exhibiting characteristic discontinuities at the collision boundary.

For exoskeleton operation, these results indicate that **impulse timing can be reliably synchronized with gait** when operating sufficiently inside the impact regime.

4.3 Basin of Attraction Analysis

Basins of attraction were computed for a representative excitation ratio within the multistable region. Two dominant attractors were observed:

- a non-impact periodic attractor,
- an impact periodic attractor.

The basin boundary exhibited a smooth but nonlinear structure, indicating moderate sensitivity to initialization.

From a device perspective, this implies that **startup conditions (e.g., initial striker position when the device powers on)** can determine whether therapeutic impulses are delivered. Implementing a short initialization routine that drives the actuator into the desired attractor can ensure consistent operation.

4.4 Impulse Consistency and Energy Consumption

Cycle-to-cycle impulse magnitude showed low variance (<5%) within stable impact regimes, indicating good repeatability for therapeutic stimulation.

Energy analysis revealed:

- Non-impact regime → lowest energy consumption
- Impact regime → moderate increase due to restitution losses

- Viscous friction case → highest dissipation

These findings suggest a trade-off between **therapeutic intensity and battery life**, with viscous-like interfaces requiring higher power to maintain equivalent impulse levels.

4.5 Implications for Gait Assistance

Mapping actuator frequency to gait phases indicates:

- 2–3 Hz → suitable for slow rehabilitation walking
- 3–5 Hz → optimal for normal cadence assistance
- 6 Hz → risk of entering non-impact regime depending on friction

Therefore, controller tuning should maintain operation within the **centre of the impact region** to avoid unintended regime switching during gait variability.

V. DISCUSSION: CLINICAL TRANSLATIONAL AND SAFETY CONSIDERATIONS

5.1 Relevance to Rehabilitation Outcomes

The results demonstrate that vibro-impact actuation can generate controlled, repeatable impulses within physiologically relevant force ranges. Such stimuli are associated with:

- Enhanced proprioceptive feedback,
- Improved neuromuscular activation,
- Facilitation of motor relearning.

The presence of distinct dynamic regimes also enables adaptive therapy, where impulse intensity could be modulated by adjusting excitation frequency rather than mechanical hardware.

5.2 Safety and Comfort

Safety is a primary concern when introducing impact-based mechanisms in wearable devices. Several findings inform safe design:

1. Predictable Impact Regimes Operating away from regime boundaries minimizes unexpected transitions and ensures consistent force delivery.
2. Role of Compliance Increased damping (viscous friction) reduces peak accelerations but increases variability, suggesting that a balanced compliance level is preferable.
3. Force Limitation via Mechanical Stop the restitution coefficient and stop position effectively cap impulse magnitude, providing a passive safety layer independent of control electronics.

4. Startup Initialization Ensuring the actuator converges to the intended attractor prevents unintended impulses when the device is donned.

5.3 Control Strategy Implications

Because multistability exists, purely open-loop frequency control may be insufficient. Clinically deployable systems should incorporate:

- Closed-loop sensing (position or acceleration),
- Regime detection algorithms,
- Soft-start routines to avoid abrupt impulses.

These measures can improve reliability in real-world rehabilitation scenarios where gait patterns vary.

5.4 Limitations

Several simplifying assumptions should be acknowledged:

- Ideal excitation neglects motor–load coupling,
- Rigid impact model ignores soft-tissue deformation dynamics,
- Single-DOF representation omits multi-joint interactions.

Future work should incorporate human–device biomechanical coupling and experimental validation on a prototype exoskeleton.

5.5 Future Directions

Potential extensions include:

- Adaptive frequency control based on gait phase detection,
- Patient-specific parameter identification,
- Integration with neuromuscular stimulation systems,
- Clinical trials evaluating functional outcomes.

DECLARATIONS

Competing Interests

The authors declare no competing interests.

Data Availability

Simulation data are available from the corresponding author upon request.

ACKNOWLEDGEMENT

The authors can acknowledge any person/authorities in this section. This is not mandatory.

REFERENCES

- [1] Okolewski, A., & Błażejczyk-Okolewska, B. Hard versus soft impact modelling of vibro-impact systems. *Nonlinear Dynamics*, 2021.
- [2] Liu, Y., Pavlovskaja, E., Hendry, D., & Wiercigroch, M. Vibro-impact responses of capsule systems. *International Journal of Mechanical Sciences*, 2013.
- [3] Liu, Y., Wiercigroch, M., Pavlovskaja, E., & Yu, H. Modelling of a vibro-impact capsule system. *International Journal of Mechanical Sciences*, 2013.
- [4] Nguyen, K. T. et al. Effect of friction on vibro-impact locomotion. *Meccanica*, 2021.
- [5] Sawicki, G. S., & Ferris, D. P. Powered ankle exoskeletons reveal neuromechanics of walking. *Journal of Experimental Biology*, 2008.
- [6] Dollar, A. M., & Herr, H. Lower extremity exoskeletons and active orthoses. *IEEE Transactions on Robotics*, 2008.
- [7] Awad, L. N. et al. A soft robotic exosuit improves walking after stroke. *Science Translational Medicine*, 2017.

AUTHORS BIOGRAPHY



Zlata Jelačić is Associate Professor in Department of Mechanics at the Faculty of Mechanical Engineering, University of Sarajevo, Bosnia and Herzegovina. PhD thesis was in the field of rehabilitation robotics, namely the development of an active above-knee prosthesis with actuated knee and ankle joints.



Maximilian Jelačić is a high-school student who has several awards in science, physics and mathematics. He won gold medal in International Science Project Olympiad, is national champion in physics and science. He qualified to participate in IPhO 2023 in Japan as the youngest participant. His hobbies are electronics and chemistry.

Citation of this Article:

Zlata Jelačić, & Maximilian Aaron Jelačić. (2026). From Nonlinear Dynamics to Neurorehabilitation: Translational Modeling of a Vibro-Impact Actuator for Ankle Exoskeletons. *International Research Journal of Innovations in Engineering and Technology - IRJIET*, 10(3), 124-129. Article DOI <https://doi.org/10.47001/IRJIET/2026.103016>
