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Determining the Coordinates of the Points on the Contours of the Development of Quadratic Surfaces and Their Intersections

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***Abstract* - The article presents a method for determining the coordinates of points on the contour of the development of quadratic surfaces and intersections on quadratic surfaces based on the method of analysis, graphic geometry combined with computational programming. The calculation results are programmed in the Autolisp language to determine the coordinates of points on the intersection, thereby constructing the development of quadratic surfaces and their intersections, applied in mechanic and teaching.**

Keywords: Intersection of quadratic surface, development of quadratic surface, development of cylindrical, development of cone, intersection of cylindrical, intersection of cone.

I. INTRODUCTION

In the world today, the application of information technology to automation in cutting and machining details has developed at a high level. In Vietnam although the mechanical engineering has made great progress in recent years, in many places technological equipment is still outdate and not suitable for modern technology. In addition, to get these cutting automatic software we have to spend a lot of money, we don't know if they are suitable and synchronized with our economic conditions and mechanical equipment or not. Small manufacturing facilities in Vietnam also don't have enough potential to purchase foreign software and equipment. Therefore, creating low-cost automatic software in cutting and machining details, which is researched to base on existing equipment conditions, suitable for worker qualification is very necessary in Vietnam.

Based on that practical need, the article builds an algorithm to determine the coordinates of a shape that develops some common quadratic surfaces in mechanical engineering. From there, we can master the database and create software that automatically develops quadratic surfaces.

Exporting the coordinates of points belonging to the developed contour can be applied in teaching and mechanical processing such as automatic cutting of shapes and welding of steel billet intersections.

1.1 What is the development of quadratic surface?

Developing a curved surface is to spread it out on a flat surface so that there are no tears or folds and use this as a cutting pattern for a flat sheet. [1], [2], [4], [6]

1.2 General computational principles of development [1], [3], [4], [6]

The calculation formula for the development is built from Top view, Front View, Right view and Left view of quadratic surface.

The calculation for development does not consider the thickness of the flat metal plates. We take the average layer of them to calculate.

When a quadratic surface is approximated, it can be replaced by many inscribed polyhedral.

II. CALCULATING DEVELOPMENTS OF SEVERAL QUADRATIC SURFACES AND THEIR INTERSECTIONS

2.1 Calculating development of quadratic surface

*** Calculating development of a cylinder [1], [4]**

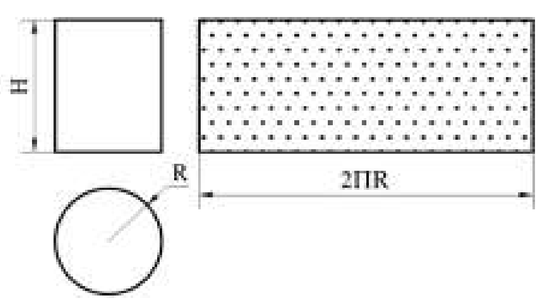

Figure 1: Development of cylinders

A cylinder with radius R and height H is developed by a rectangular of height H and length 2πR. (1)

*** Calculating development of cones [1], [2], [3], [6]**

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Given a right circular cone with the radius of its base R, height H, slant height Land angle α.

The development of cone is a circular sector obtained by unfolding the surface of one nappe of the cone.

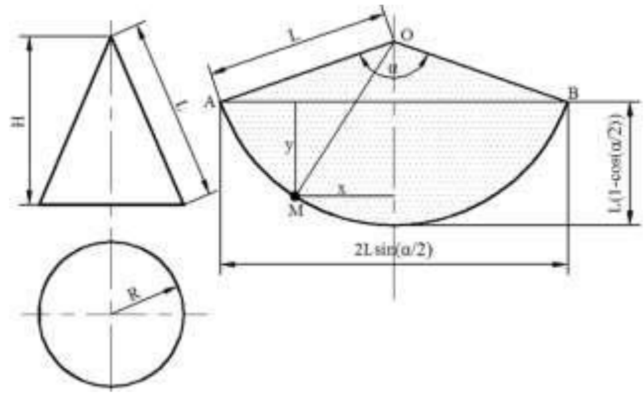

Figure 2: Development of a cone

The slant height of the cone is$L = \sqrt{H^2 + R^2}$ (2)

$$L.\alpha = 2\Pi R \rightarrow \alpha = \frac{2\Pi R}{L} (3)$$

The central angle α in radians is calculated as follows:

The coodinates of M on the circular sector are determined as shown in Figure 2. We have:

$$y = \sqrt{L^2 - x^2} - L\cos(\alpha/2) \ \ (4)$$

In case of frustum of a cone, its development is calculated similarly.The small base of the cone will be developed like the large base. Given a frustum of a cone whose height is H, radius of the small base is R1, the large base is R. The small base of the cone will be developed like the large base.

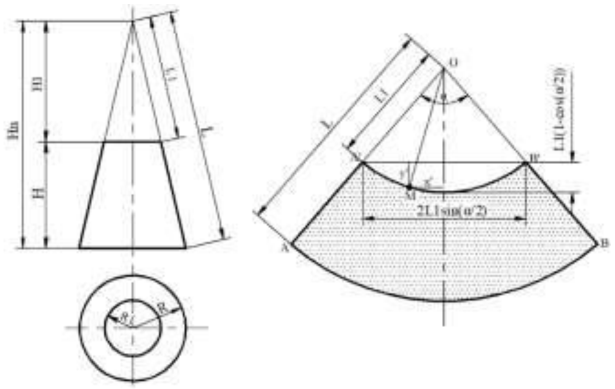

Figure 3: Development frustum of a cone

$$\frac{H_1}{H_n} = \frac{R_1}{R} \rightarrow \frac{H_1}{H + H_1} = \frac{R_1}{R}$$

$$\rightarrow H_1R = R(H + H_1) \rightarrow H_1 = \frac{R_1H}{R - R_1}$$

$$H_n = H_1 + H \ (5)$$

$$L_1 = \sqrt{H_1^2 + R_1^2} (6)$$

$$L = \sqrt{H_n^2 + R^2} \ (7)$$

The x', y' coordinates of the small base are determined as shown.

$$y' = \sqrt{L_1^2 - x'^2} - L_1\cos\left(\frac{\alpha}{2}\right) \ \ (8)$$

*** Calculating development of a sphere [1], [3]**

In the development of spheres an approximate method is used. In this method the sphere is cut into equal number of meridian sections or lunes which can be considered as sections of cylinders.

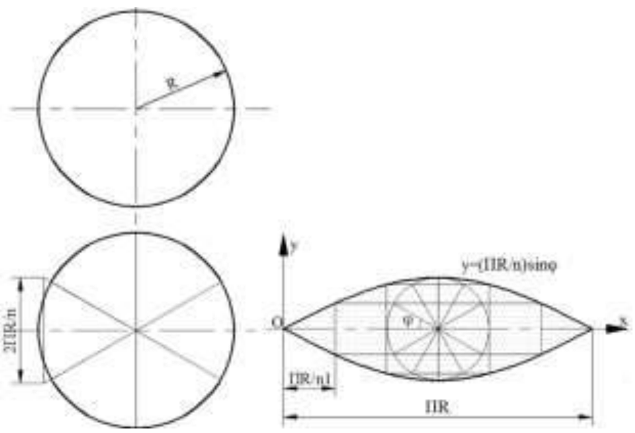

Figure 4: Development of a sphere

To develop the sphere the meridians are used to divide it into n equal sections whose equatorial arc AB = 2πR/n. This section is replaced by cylindricalpieces bounded by two meridians. So, the height of the segment on the developed figure is πR and its width is 2πR/n. Let angle φ change from 0 to 180 degrees corresponding to kφ, we havethe coordinates of the contour of the developed figure as:

$$x = \frac{\Pi R}{180}k\varphi; y = \frac{\Pi R}{n}\sin\varphi \ (9)$$

2.4 Calculating development ofseveralintersections of quadratic surfaces

*** Calculating the development of acylinder cut by a plane inclined at an angle α [1], [2], [3], [4], [5], [7]**

The inclined plane intersects the cylinder along an ellipse.If the plane is inclined at an angle α to the x-axis, the

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envelope of the development is a sinusoid with step length 2πR and amplitudeRtgα. When the angle φchanges from 0 to 90 degrees the circle is divided into n equal small angles kφ. We have the coordinates of the contour of the developed figure as:

$$x = \frac{\Pi R.k\varphi}{180}; y = R.tg\alpha.\sin\varphi \quad (10)$$

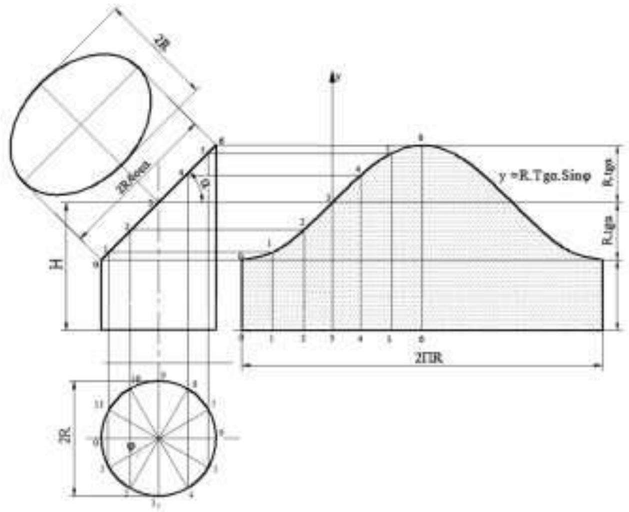

Figure 5: Development of cylinder cut by an inclined plane

*** Calculating development of two two consecutive cylinders[1], [2], [3], [4], [5]**

Two consecutive cylinders have the same radius R and their intersection is an ellipse developed as shown in the figure 6.This ellipse is inclined an angle α. The envelope of the intersection is a sinusoid with step length 2πR and amplitude RTgα.

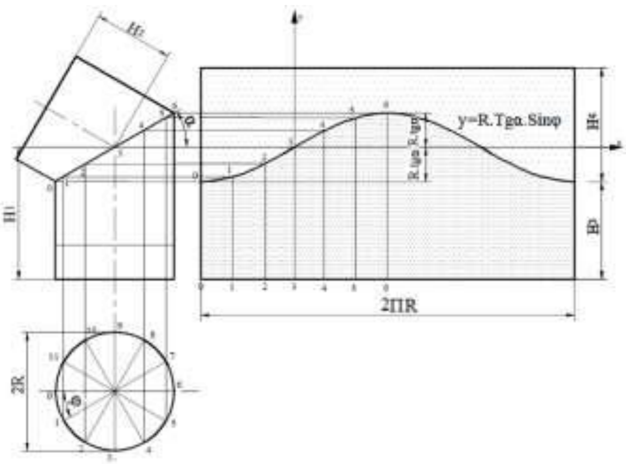

Figure 6: Development of two consecutivecylinders

When the angle φchanges from 0 to 90 degrees the circle is divided into n equal small angles kφ. We have:

$$H_3 = H_1 - R.tg\alpha\ ; \quad H_4 = H_2 + R.tg\alpha$$

The coordinates of the contour of the developed are:

$$x = \frac{\Pi R.k\varphi}{180};\ y = R.tg\alpha.\sin\varphi\ (11)$$

*** Calculating development of two cylindersintersecting at angle α [1]**

Two intersectingcylinderswith the different radius have their intersection isa quartic curve.The coordinates of K on the developed intersection are calculated as figure 7.

- *Calculating development of the first cylinder:*

According to the figure

$$xk = \frac{\Pi R_1.k\varphi}{180}\ (12)$$

Set KB=j_k và BC=i_k→ $y_k = i_k + j_k + H_4$(13)

i_k= BC= CN.cotgα = (R_1+ R_1cos$k\varphi$)cotgα

= R_1(1+cos$k\varphi$)cotgα

$$j_k = KB = \frac{BE}{\sin\alpha} = \frac{JH}{\sin\alpha} = \frac{R-IH}{\sin\alpha}$$

$$= \frac{(R - \sqrt{R^2 - R_1^2\sin^2 k\phi})}{\sin\alpha}$$

$$H_4 = H_1 - \frac{R}{\sin\alpha} - \frac{R_1}{tg\alpha}$$

The width of the first cylinder development is $2\pi R_1$

- *Calculating development of the second cylinder:*

The second cylinder with radius R and height H is developed by a retangular of heigh H and length 2πR. When dividing the base of the first cylinder into equal angles kφ, the base of the second cylinder is devided into equal angles kγ. The relationship between φ and γ is:

$$R_1\sin k\varphi = R\sin k\gamma \rightarrow \sin k\gamma = \frac{R_1}{R}\sin k\varphi$$

$$x'_k = \frac{\Pi R.k\gamma}{180}(14)$$

$$y'k = \frac{1}{\sin\alpha}[R(1-\cos k\gamma)\cos\alpha + R_1(1+\cos k\varphi)](15)$$

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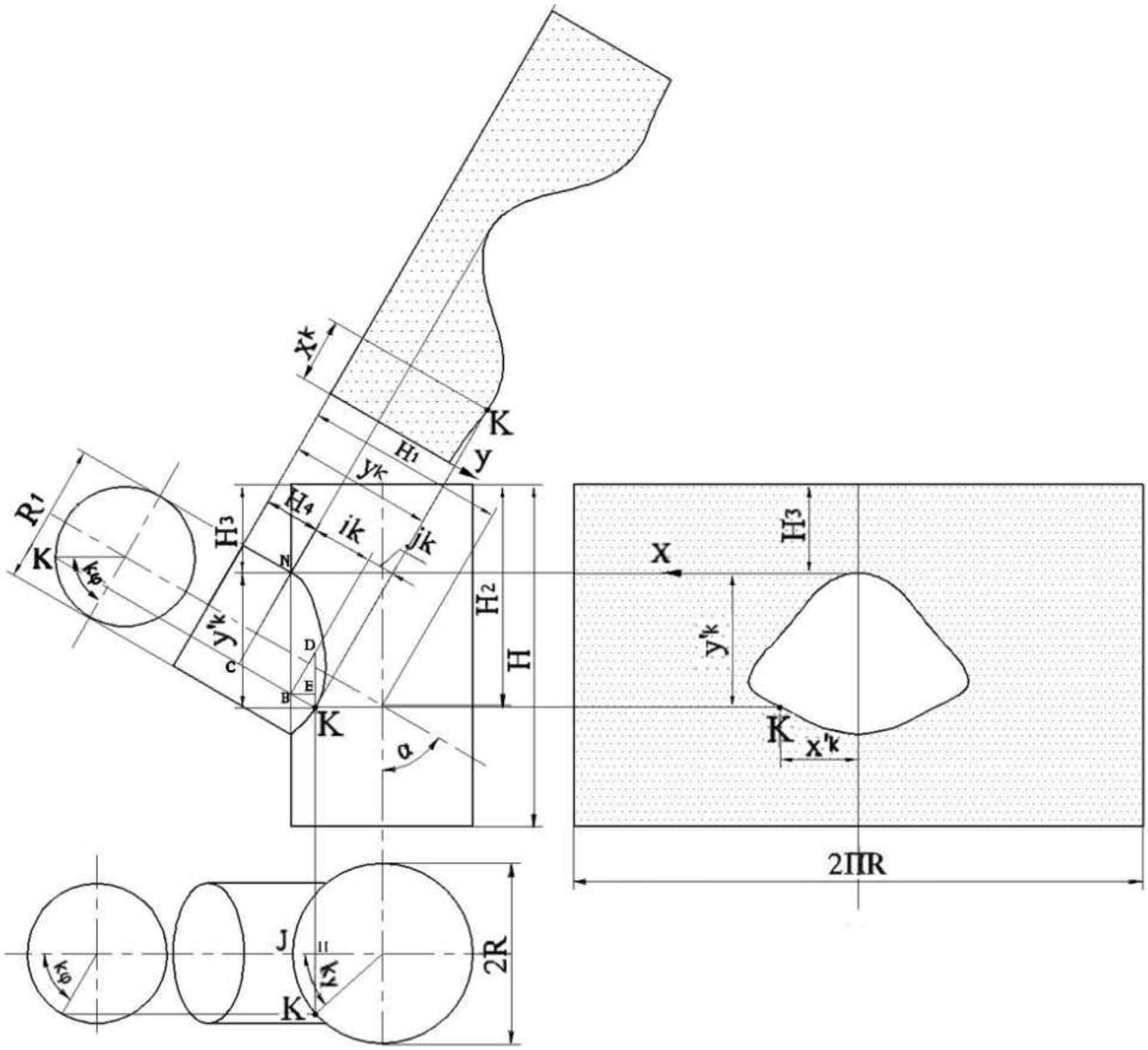

Figure 7: Development of two cylinders

*** Calculating the development of a cone cut by a plane inclined at an angle α [1], [2], [3]**

The intersection of cone cut by an inclined plane as shown in the figure 8 is an ellipse.First we develope the cone as above.Then the coordinates of the points of the ellipse development are calculated according to polar coordinate. The distance from the top of the cone to the intersection point along the length of the slant is:

$$p = \frac{\sin\alpha - tg\beta.\cos\alpha}{\sin\alpha - tg\beta.\cos\varphi\,.\cos\alpha} L \ (16)$$

where L is the slant length$L = \sqrt{R^2 + H^2}$and

$$Tg\beta = \frac{R}{H}$$

When the angle φchanges from 0 to 90 degrees, the coordinates of the points in the ellipse development are determined.

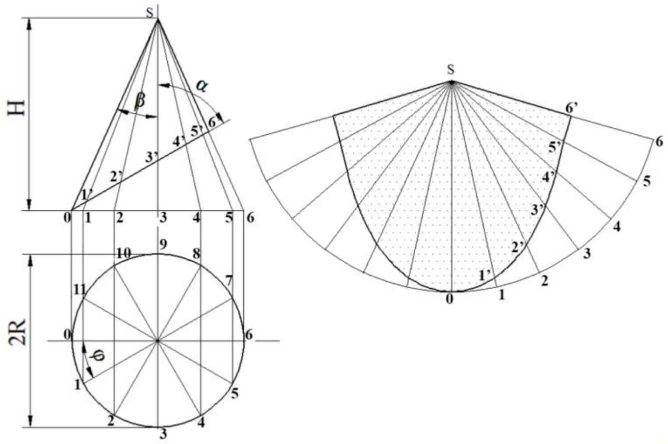

Figure 8: Development of a cone cut by a plane inclined at an angle α

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III. RESULTS AND DISCUSSIONS

Based on the calculation of the coordinates of the developments as above, the Autolisp programming language is used to automatically determine the coordinates and draw the developments.

3.1 The results of the development of quadratic surfaces

From formulas (1) and the AutoLips programming language, we can get can obtain the cylindrical development result as shown in Figure 9.

Figure 9: The cylindrical development result

The cone development resultsare also automatically drew from the formulas (2), (3), (4), (5), (6), (7), (8) as shown in Figure 10 and Figure 11.

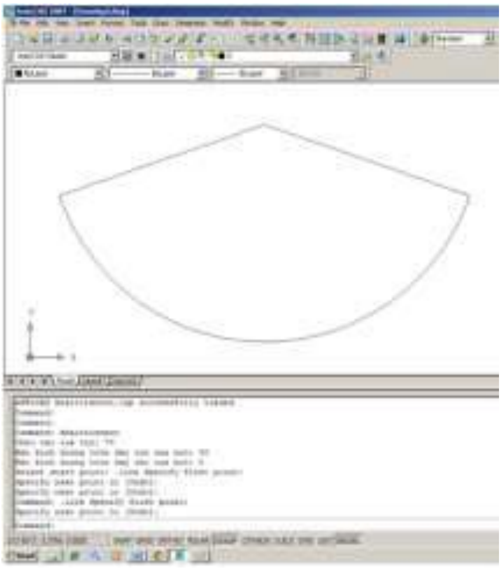

Figure 10: The cone development result

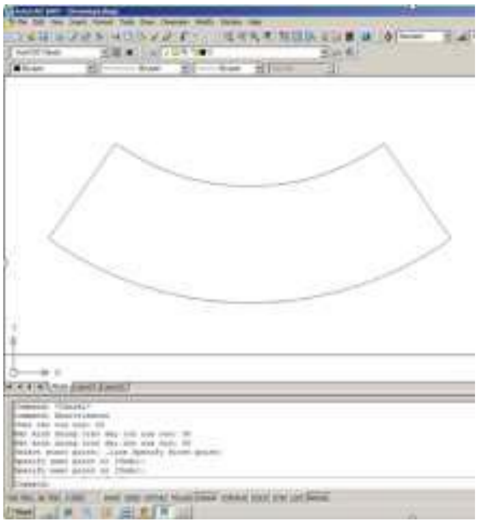

Figure 11: The frustum of a cone development result

Applying formula (9), we have the following result of sphere development:

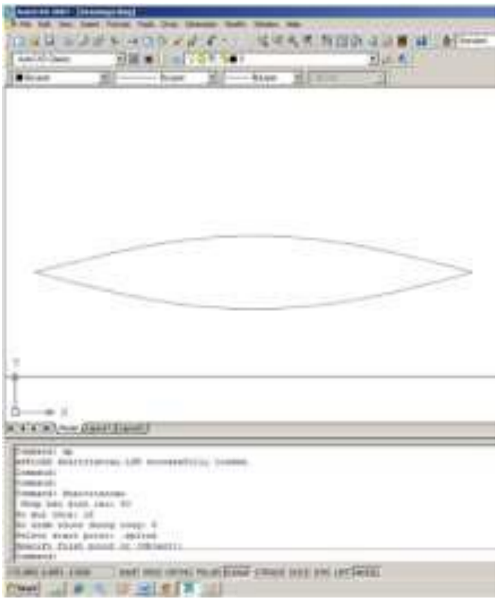

Figure 12: The spheredevelopment result

3.1 The results of the development of intersected quadratic surfaces

Base on formulas (10) and Autolips programming language, we have the following result of development of cylinder cut by a plane inclinedas shown in Figure 13.

The developments of two consecutive cylindersare also automatically drew from the formulas (2), (3), (4), (5), (6), (7), (8) as shown in Figure 14.

Applying formulas (12), (13) and Autolips programming language, we have the following result of development of two cylinders intersectingas shown in Figure 15.

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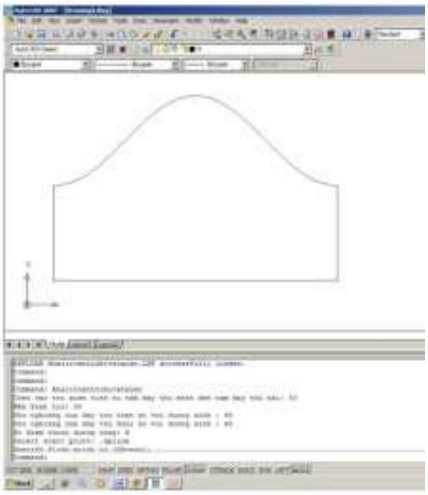


Figure 13: The development of cylinder cut by a plane inclined result

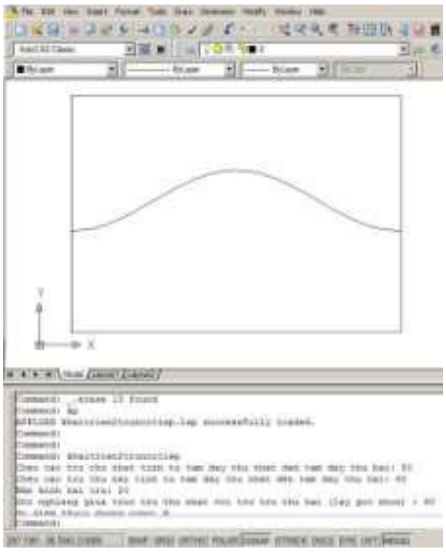


Figure 14: The development of two consecutive cylinders result

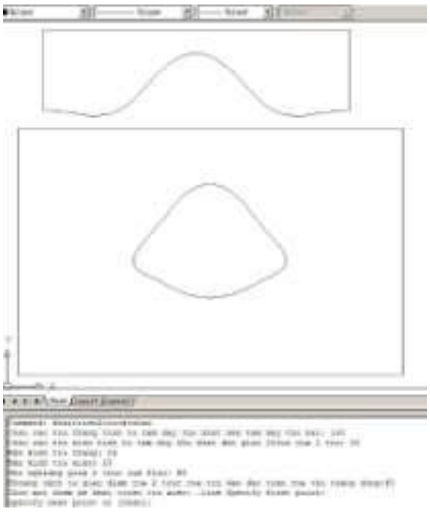


Figure 15: The development of the intersections of two cylinders

The cone cut by a plane inclined is developed by formulas (16) as shown Figure 16.

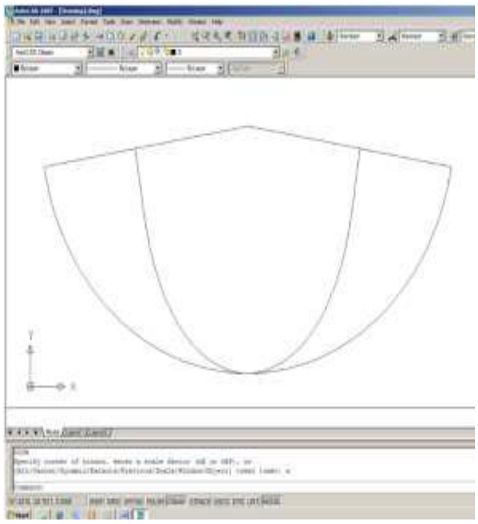
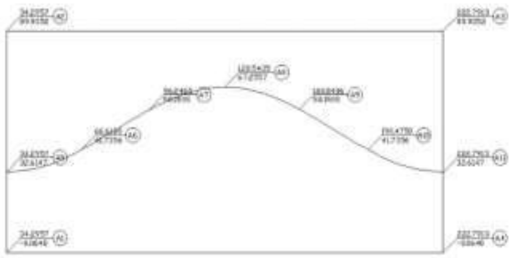


Figure 16: The development of cone cut by a plane inclined result

After the quadratic surfaces and their intersections are developed, the coordinates will be displayed and listed on the screen.



STT	Toa do X	Toa do Y
A1	34.2957	-0.0648
A2	34.2957	89.9352
A3	222.7913	89.9352
A4	222.7913	-0.0648
A5	34.2957	32.6147
A6	66.6120	41.7356
A7	96.2465	58.1505
A8	128.5435	67.2557
A9	160.8406	58.1505
A10	190.4750	41.7356
A11	222.7913	32.6147

Figure 17: The coordinates are displayed and listed on the screen.

Exporting the coordinates of points belonging to the development contour helps to master the database that can be applied in teaching, in mechanical processing and built automatical development software.

IV. CONCLUSION

In short, this paper demonstrates the method of determining the coordinates of the points on the development of quadratic surfaces and their intersections, using Autolisp programming language to automatical find the coordinates of these points and draw the development. This helps to master the database to build software to control the automatic mechanical processing system, which is suitable for the equipment conditions as well as the qualifications of

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Vietnamese workers. It can be applied in teaching and mechanical processing such as automatic cutting of shapes and welding of steel billet intersections.

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