

Proposed Equation of Maximum Principal Stress for Feedwater Heater Tubes with Circumferential Hoop Flaw Defect

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Abstract - Feedwater heater tubes with local wall-thinning defects are often found in the heat exchangers of thermal power plants. One of the most typical local wall-thinning shapes is circumferential hoop flaw defect. Determining the maximum stress acting on the tube with a circumferential defect is essential to ensure the safe operation of the heat exchange system before the tube can be failure due to stress concentration at the local thinning. This paper introduces a theoretical and numerical analysis of the maximum principal stress on the feedwater heater tubes with a circumferential hoop flaw defect under pressure and thermal loadings. The effect of three dimensions of this defect on the maximum principal stress is also studied. Using finite element analysis and regression, analytic equation for the maximum principal stress on the inner surface of a tube with a circumferential hoop flaw defect is proposed. It is found that the maximum principal stress occurs on the inner surface of the tube. The depth of the defect is the main factor influencing the maximum principal stress. The longitudinal length and the circumferential angle parameter of the defect also have effect on the maximum principal stress.

Keywords: Maximum principal stress; Feedwater heater tubes; Circumferential defect; Finite element method; Regression equation.

I. INTRODUCTION

Feedwater heater tubes are critical component in thermal power systems. The primary function of the tube is to transfer heat from high-pressure steam outside tube to the feedwater inside. This process increases the thermodynamic efficiency of the plant and significantly reduces fuel consumption. These tubes are typically made from heat-resistant, high-pressure, and corrosion-resistant steel alloys. The tubes operate under harsh conditions such as high temperatures and pressures. The performance and condition of the feedwater heater tubes have a direct impact on the overall efficiency and safety of the power plant. Therefore, regular monitoring, inspection, and

maintenance of these tubes are essential to prevent failures and to extend the service life of the equipment.

Evaluating the maximum stress of the tube is essential to ensure the safe operation of the equipment, especially when local thinning defect or wear corrosion occurs on the tube. An experimental and numerical analysis of the plastic limit loads in Inconel 690 steam generator tubes with various local wall-thinning defects have been conducted [1]. They introduced three typical types of local wall-thinning defect of the tube. There has also been research on the effect of the dimensions of local wall-thinning defects of the tube on stresses. Determination of stresses such as hoop stress, radial stress for high-pressure feedwater heater tubes with outer thinning (either uniform or eccentric thinning) under pressure and thermal loading have also been analyzed [2]. Plugging criteria and stress determination for high-pressure feedwater heater tubes with local wall-thinning was introduced [3]. Their results focus on one type of local wall-thinning shape: arc-like flaw defect). It also suggested the regression equations for hoop stress and axial stress of the tube.

The aim of the research is to determine the maximum principal stress within feedwater heater tubes with a circumferential hoop flaw defect using finite element method (FEM) and regression equation analysis. The effect of three dimensions of circumferential defect on maximum principal stress is also analyzed. Using regression method, analytic equations for the maximum principal stress on the inner surface of the tube with a circumferential defect are proposed. In this study, comparing the proposed equation with the results of finite element analysis of the maximum principal stress is conducted.

II. FINITE ELEMENT ANALYSIS FOR THE TUBE WITH A LOCAL THINNING

This study focused on the straight section of a U-tube in the de-superheating zone, which is the most dangerous location to failure due to the highest temperature difference between the inner and outer surfaces of the tube. Modeling of

the tube with a circumferential hoop flow defect was shown in the Figure 1.

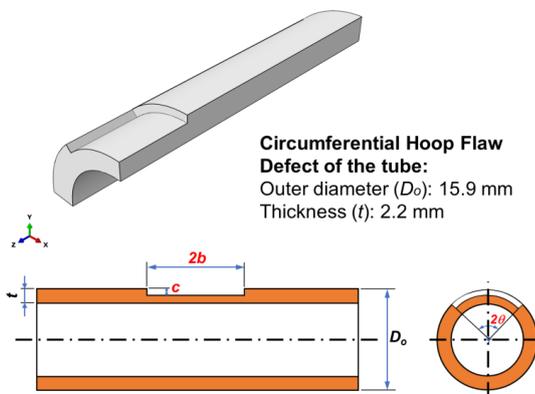


Figure 1: Modeling of the tube with a circumferential hoop flow defect

2.1 Finite element models and effect of three dimensions of circumferential defect on maximum principal stress

a) Finite element models

Finite element analysis (FEA) was conducted using ABAQUS software (version 2019) to determine the mechanical and thermal stresses on the tube with circumferential hoop flow defect. A half model of the tube was studied to take advantage of symmetry, and the internal pressure, external pressure, and temperature gradient were analyzed. An axial uniform load was applied to one the end of the tube to represent the thrust of pressure, while the other end of the tube was modeled using symmetry plane-z. Also, the plane-x was applied to the symmetry plane condition.

Figure 2 presents further details regarding the loads, boundary conditions, and meshing of the tube with a circumferential hoop flow defect. Mesh independence was confirmed before using the FE models (100 models) ranges from 100,000 to 200,000 elements to make sure the convergence of the results.

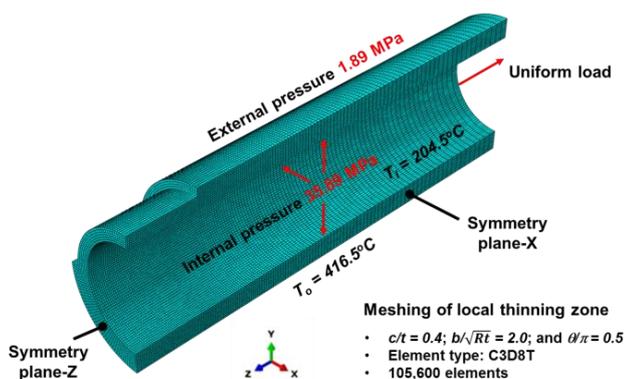


Figure 2: Loads, boundary conditions, and meshing of the tube with a circumferential hoop flow defect

The tubes for the feedwater heater were fabricated from seamless stainless SA-213 Grade TP304N material. The mechanical properties of this material can be found in the ASME Code, Section II, Part D [4]. Other parameters such as Young’s modulus (E), thermal conductivity (k), the thermal expansion coefficient (α), yield strength (σ_{YS}), and ultimate tensile strength (σ_{UTS}) are presented in Table 1, as calculated based on the average of the inner and outer temperatures of the tube (at 235 °C).

The length of the feedwater heater tube models with defect is selected to be 50 mm. The maximum defect length studied in FEA is 11.64 mm. The distance from the boundary of the local wall-thinning defect to discontinuity zone (another end of tube) is equal to $50 - 11.64 = 38.36$ mm, which also exceeds $2.5\sqrt{Rt}$ (9.70 mm). The influences of the tube length models can be ignored.

Table 1: Material properties of SA-213 Grade TP304N at 235 °C

Material properties	SA-213 Grade TP304N
Young’s modulus (GPa)	175
Poisson’s ratio	0.31
Density (kg/m ³)	8100
Thermal conductivity (W/m.°C)	19.6
Thermal expansion coefficient (°C ⁻¹)	17.8×10^{-6}
Yield strength (MPa)	149
Ultimate tensile strength (MPa)	497

b) Effect of three dimensions of circumferential defect on maximum principal stress

Using FEA, the stresses of a tube with circumferential hoop flow are assessed for various values of c (0.22, 0.44, 0.66, 0.88, 1.10 mm), b (3.88–11.64 mm), and θ (45° – 180°), where c , b , and θ are the dimensions of the circumferential hoop flow defect defined in Figure. 1. In detail, c is the depth of thinned path of the local wall-thinning defect (circumferential hoop flow defect), b is the half-length of thinned path along axial direction, and θ is the half of circumferential angle of the local wall-thinning defect.

In order to reduce models in the FEA, the non-dimensional parameters c/t , b/\sqrt{Rt} , and θ/π were employed, where c/t is the ratio of the depth of the local wall-thinning defect to the wall thickness of the feedwater heater tube; b/\sqrt{Rt} is the ratio of the longitudinal length of the local wall-thinning defect to the square root of product of the average radius of the tube multiplies the wall thickness; and θ/π is the ratio of the half of circumferential angle of the local wall-thinning defect to π .

In order to study the effect of the depth, length, and circumferential angle of the local wall-thinning defect on the stresses, 100 variations of the FEA model are created based on different combinations of the c/t (0.1, 0.2, 0.3, 0.4, 0.5), b/\sqrt{Rt} (1.0, 1.5, 2.0, 2.5, 3.0) and θ/π (0.25, 0.50, 0.75, 1.00) ratios.

2.2 Analytic equation for maximum principal stress in the absence of circumferential defect

This study, the model tube is subjected to both internal and external pressure. The maximum principal stress (or hoop stress) equation as a function of the radius [7] was given by Eq. (1).

$$\sigma_{1,p} = \sigma_{\theta,p} = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} + \frac{(P_i - P_o) r_i^2 r_o^2}{(r_o^2 - r_i^2) r^2} \quad (1)$$

The maximum principal stress equation due to thermal loading was determined using Eq. (2). Where, T_o^* and T_i^* are the temperature on the outer surface and the inner surface of the tube, respectively.

$$\sigma_{1,th} = \sigma_{\theta,th} = \frac{E\alpha(T_o^* - T_i^*)}{2\ln(r_o/r_i)} \left[\frac{r_o^2}{r_o^2 - r_i^2} \left(1 + \frac{r_i^2}{r^2} \right) \ln\left(\frac{r_o}{r_i}\right) - \ln\left(\frac{r}{r_i}\right) - 1 \right] \quad (2)$$

T is the temperature distribution inside the wall of the tube.

The maximum principal stress due to combination between pressure and thermal loading was determined by superposing all of the stresses in the hoop directions.

$$\sigma_1 = \sigma_\theta = \sigma_{1,p} + \sigma_{1,th} \quad (3)$$

2.3 Proposed equation for maximum principal stress in the presence of circumferential defect

To account for the presence of a local wall-thinning defect in the tube, the maximum principal stress equation for the inner surface must be modified:

$$\sigma_1 = \left\{ \frac{(P_i r_i^2 - P_o r_o^2) + (P_i - P_o) r_o^2}{r_o^2 - r_i^2} + \frac{E\alpha(T_o^* - T_i^*)}{2\ln(r_o/r_i)} \left[\frac{2r_o^2}{r_o^2 - r_i^2} \ln\left(\frac{r_o}{r_i}\right) - 1 \right] \right\} \times F_e\left(\frac{c}{t}; \frac{b}{\sqrt{Rt}}; \frac{\theta}{\pi}\right) \quad (4)$$

Where $F_e(c/t; b/\sqrt{Rt}; \theta/\pi)$ is the maximum principal stress correction function for local thinning which depends on the parameters c/t (0.1–0.5), b/\sqrt{Rt} (1.0–3.0), and θ/π (0.25–1.00).

The expression for F_e was obtained from the finite element solution for specific values of the c/t , b/\sqrt{Rt} , and θ/π ratios.

$$F_e\left(\frac{c}{t}; \frac{b}{\sqrt{Rt}}; \frac{\theta}{\pi}\right) = \frac{\sigma_{1_FEM}}{\frac{(P_i r_i^2 - P_o r_o^2) + (P_i - P_o) r_o^2}{r_o^2 - r_i^2} + \frac{E\alpha(T_o^* - T_i^*)}{2\ln(r_o/r_i)} \left[\frac{2r_o^2}{r_o^2 - r_i^2} \ln\left(\frac{r_o}{r_i}\right) - 1 \right]} \quad (5)$$

The interpolated function for F_e was developed based on response surface modeling (RSM). RSM postulates a model of the form [8]

$$y(x) = f(x) + \varepsilon \quad (6)$$

Where, $y(x)$ is the unknown function of interest, $f(x)$ is the polynomial approximation of x , and ε is random error. A second-order model was employed for optimization of the response y .

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \beta_{ij} x_i x_j + \varepsilon \quad (7)$$

The parameters β_0 , β_i , β_{ii} , and β_{ij} , of the polynomials in Eq. (7) were determined using least-squares regression, which minimizes the sum of the squares of the difference between the predicted and actual values. The coefficients of Eq. (7) were found using Eq. (8):

$$\beta = [X'X]^{-1} X'y \quad (8)$$

Where, X is the design matrix of sample data points, X' is its transpose, and y is a column vector that contains the values of the response at each sample point. The second-order response surface model for the correction function F_e was fit to the 100 sample points using ordinary least-squares regression. F_e is given by Eq. (9):

$$F_e\left(\frac{c}{t}; \frac{b}{\sqrt{Rt}}; \frac{\theta}{\pi}\right) = F_e(x_1; x_2; x_3) = 1.0588 - 0.2413x_1 + 0.0192x_2 + 0.1596x_3 + 1.8664x_1^2 - 0.0054x_2^2 - 0.0968x_3^2 + 0.0405x_1x_2 - 0.0066x_2x_3 - 0.2773x_1x_3 \quad (9)$$

Where, $x_1 = \frac{c}{t}$; $x_2 = \frac{b}{\sqrt{Rt}}$; $x_3 = \frac{\theta}{\pi}$

III. RESULTS AND DISCUSSIONS

Figure 3 shows finite element method results for Von Mises and maximum principal stress of the tube. Maximum principal stress takes place at inner surface of the tube for a circumferential hoop flow defect. The result indicates that the maximum principal stress has a value approximately equal to Von Mises stress.

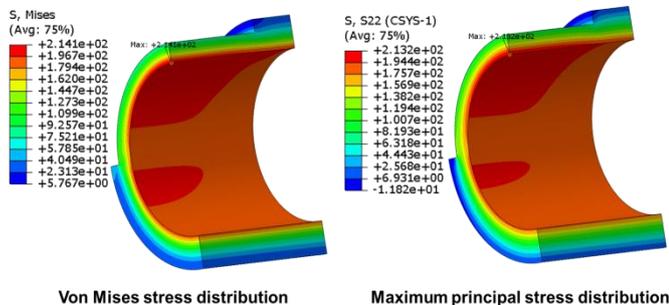


Figure 3: FEM results for Von Mises and maximum principal stress

Figure 4 compares the results of F_e correction function between finite element analysis and the proposed equation. The results obtained from the proposed equation match with the finite element results, with an average difference of less than 5%.

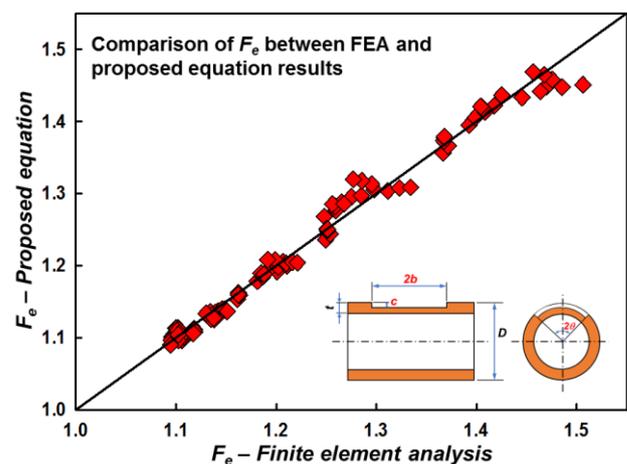


Figure 4: Comparison of correction function (F_e) between FE analysis and proposed equation results

Comparison of the proposed equation and finite element analysis results at $\theta/\pi = 0.5$ are listed in Table 2. In this study, the wall thickness and outer diameter of the tube are constant ($t/D = 0.139$ and $t = 2.2$ mm). In order to study the effect of the depth (c) and the longitudinal length (b) of the circumferential hoop flow defect on the maximum principal stress, a fixed value of the circumferential angle $\theta/\pi = 0.5$ is

employed. The results show that maximum discrepancy of the maximum principal stress between the FEA results and those calculated using the proposed equation is -3.67% .

Table 2: Comparison of the proposed equation and FEA results at $\theta/\pi = 0.5$

c/t ratio	b/\sqrt{Rt} ratio	Maximum principal stress		
		FEA (MPa)	Proposed equation (MPa)	Discrepancy (%)
0.1	1.0	178.9	180.4	0.84
0.1	1.5	179.0	180.9	1.06
0.1	2.0	179.1	181.0	1.06
0.1	2.5	179.3	180.6	0.73
0.1	3.0	179.4	179.9	0.28
0.2	1.0	184.9	184.0	-0.49
0.2	1.5	185.7	184.8	-0.48
0.2	2.0	186.3	185.2	-0.59
0.2	2.5	186.7	185.2	-0.80
0.2	3.0	187.1	184.8	-1.23
0.3	1.0	193.5	193.6	0.05
0.3	1.5	195.4	194.8	-0.31
0.3	2.0	196.7	195.6	-0.56
0.3	2.5	197.7	195.9	-0.91
0.3	3.0	198.5	195.7	-1.41
0.4	1.0	205.7	209.4	1.80
0.4	1.5	208.9	210.9	0.96
0.4	2.0	213.2	211.9	-0.61
0.4	2.5	215.1	212.6	-1.16
0.4	3.0	216.9	212.8	-1.89
0.5	1.0	230.5	231.1	0.26
0.5	1.5	235.0	233.0	-0.85
0.5	2.0	238.0	234.4	-1.51
0.5	2.5	241.5	235.3	-2.57
0.5	3.0	244.9	235.9	-3.67

The results analysis by finite element method of maximum principal stress at $\theta/\pi = 0.5$ are shown in Figure 5. It can be seen from Figure 5 that as the c/t ratio parameter increase, the maximum principal stress tends to increase significantly. At the same value of c/t ratio, the maximum principal stress will increase slightly when b/\sqrt{Rt} ratio parameter increase. It is clear that the defect depth (c) of the circumferential hoop flow defect is the main factor influencing the maximum principal stress. Defect longitudinal length (b) is

an important factor influencing the maximum principal stress. Also, the circumferential angle (θ) of the tube defect has effect on the stress.

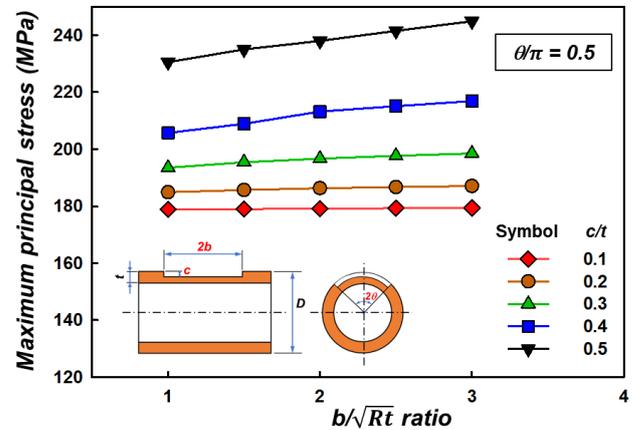


Figure 5: FEA results of maximum principal stress at $\theta/\pi = 0.5$

IV. CONCLUSION

The determination of the maximum principal stress on the inner surface of a tube with a circumferential hoop flow defect is conducted in this study. Proposed equation of the maximum principal stress for the tube with the local thinning is given based on the finite element method results and the regression equation. Also, effect of three dimensions of circumferential defect on maximum principal stress is analyzed. The key findings and results are as follows:

- The maximum principal stress occurs on the inner surface of the tube at the local thinning defect location.
- The depth of the defect is the main factor influencing the maximum principal stress.
- The longitudinal length and the circumferential angle parameter of the defect also have effect on the maximum principal stress.

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