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Convergence Results for Noor Iteration Procedure in Convex G-Metric Spaces

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Abstract **- In this paper, several convergence results are obtained for various mappings in convex G-metric spaces using Noor iterative procedure. The Results obtained generalized a variety of comparable results.**

Keywords: Convex G-metric space, fixed point, iteration, convergence.

I. INTRODUCTION AND PRELIMINARIES

Mustafa and Sims [12] defined the generalization of metric space in 2006 and named it as G-metric space. Since then, several fixed-point results for various mappings have been obtained in G-metric spaces [1-3, 7-11, 13].

Basic definitions and results:

Definition 1.1: [12]. A function $\tilde{\zeta}:\Im\times\Im\times\Im\to R^{+}$ (where $\Im\neq\phi$) is said to be a **G-metric** on $\Im$ if it satisfies:

(i) $\tilde{\zeta}(\chi,\rho,\kappa)=0$ when $\chi=\rho=\kappa$,

(ii) $0<\tilde{\zeta}(\chi,\chi,\rho)$ $\forall\chi\neq\rho$ and $\chi,\rho\in\Im$,

(iii) $\tilde{\zeta}(\chi,\chi,\rho)\leq\tilde{\zeta}(\chi,\rho,\kappa)$ $\quad\forall\rho\neq\kappa$ and $\chi,\rho,\kappa\in\Im$,

(iv) $\tilde{\zeta}(\chi,\rho,\kappa)=\tilde{\zeta}(\chi,\kappa,\rho)=\tilde{\zeta}(\rho,\kappa,\chi)=.....,$

(v) $\tilde{\zeta}(\chi,\rho,\kappa)\leq\tilde{\zeta}(\chi,\gamma,\gamma)+\tilde{\zeta}(\gamma,\rho,\kappa),\ \forall\chi,\rho,\kappa,\gamma\in\Im,$

Then, $(\mathrm{P},\tilde{\zeta})$ is said to be a **G-metric space (GM-space).**

Definition 1.2: [12]The sequence $\{\chi_a\}$ of points of $\Im$ is called **G-convergent** to ω in GM- space $(\Im,\tilde{\zeta})$ if for ach $\varepsilon>0$, $\exists\, l\in N$ such that $\tilde{\zeta}(\omega,\chi_a,\chi_b)<\varepsilon$ for all $a,b\geq l$. We can say that ω is limit of the sequence $\{\chi_a\}$.

Definition 1.3: [12] A sequence $\{\chi_a\}$ in a GM- space $(\Im,\tilde{\zeta})$ is called G-**Cauchy** if for each $\varepsilon>0$, $\exists\, l\in N$ such that $\tilde{\zeta}(\chi_a,\chi_b,\chi_c)<\varepsilon$ $\forall a,b,c\geq l$.

Definition 1.4: [12] If every G-Cauchy sequence of points of a GM- space $(\Im,\tilde{\zeta})$ is G-convergent then GM- space $(\Im,\tilde{\zeta})$ will be called **G- complete.**

Lemma 1.5: [12] Let $(\Im,\tilde{\zeta})$ be a GM-space, then $\tilde{\zeta}(\chi,\rho,\rho)\leq 2\tilde{\zeta}(\rho,\chi,\chi)$ $\forall\ \chi,\rho\in\Im$.

Takahashi [15] defined Convex metric space in 1970 and proved some fixed-point results for non expansive mappings. There after many extensions of this concept are obtained see [4-6, 14, 16, 17].

In 2019, Yildirim and Khan [18] defined convex G-metric space as follows:

Definition 1.6: [18].A function $\tilde{W}:\Im\times\Im\times\mathrm{I}\times\mathrm{I}\to\Im$ defined on aGM-space $(\Im,\tilde{\zeta})$ is said to be a **convex structure** on $(\Im,\tilde{\zeta})$ if $\tilde{\zeta}(\tilde{W}(\chi,\sigma;\varepsilon,\lambda),o,\upsilon)\leq\varepsilon\tilde{\zeta}(\chi,o,\upsilon)+\lambda\tilde{\zeta}(\sigma,o,\upsilon)$ $\forall\chi,\sigma,o,\upsilon\in\Im$ and $\varepsilon,\lambda\in\mathrm{I}=[0,1]$ satisfying $\varepsilon+\lambda=1$.Then $(\Im,\tilde{\zeta},\tilde{W})$ is said to be a **convex GM- space.**

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Definition 1.7: [15] Let $S \neq \phi$, then **S**is said to be convex subset of convex GM-space $(\Im, \tilde{\zeta}, \tilde{W})$ if $\tilde{W}(\chi,\sigma;\varepsilon,\lambda) \in S\ \forall \chi,\sigma \in S$ and $\varepsilon,\lambda \in \mathrm{I} = [0,1]$.

Then Yildirim and Khan [18] transformed the Mann iterative procedure in convex GM- space as follows:

Definition 1.8: [18] Let $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM- space and $\Upsilon : \Im \to \Im$ be a map.Let $\{i_f\}$ be any sequence on [0,1] for $f \in N$. Then for any $\tau_0 \in \Im$ and $f \in N$, the **Mann iterative procedure** is the sequence $\{\tau_f\}$ defined as

(1.1.1) $\tau_{f+1} = \tilde{W}(\tau_f, \Upsilon\tau_f; 1-i_f, i_f),$

Now, we will transform the Ishikawa and Noor iterative procedure in convex GM- space:

Definition 1.9: Let $\Upsilon : \Im \to \Im$ be a mapping on a convex GM- space $(\Im, \tilde{\zeta}, \tilde{W})$. Let $\{i_f\},\{j_f\}$ be any two sequences on [0,1] for $f \in N$. Then for any $\tau_0 \in \Im$ and $f \in N$, the **Ishikawa iterative procedure** is the sequence $\{\tau_f\}$ defined as

$$(1.1.2)\quad \begin{cases} \tau_{f+1} = \tilde{W}(\tau_f, \Upsilon\upsilon_f; 1-i_f, i_f), \\ \upsilon_f = \tilde{W}(\tau_f, \Upsilon\tau_f; 1-j_f, j_f), \end{cases}.$$

It should be noted here that if we choose $j_f = 0,$ in (1.1.2) then we get (1.1.1). Thus, Mann iterative procedure is a special case of Ishikawa iterative procedure.

Definition 1.10: Let $\Upsilon : \Im \to \Im$ be a mapping on a convex GM- space $(\Im, \tilde{\zeta}, \tilde{W})$. Let $\{i_f\},\{j_f\},\{k_f\}$ be any two sequences on [0,1] for $f \in N$. Then for any $\tau_0 \in \Im$ and $f \in N$, the **Noor iterative procedure** is the sequence $\{\tau_f\}$ defined as

$$(1.1.3)\quad \begin{cases} \tau_{f+1} = \tilde{W}(\tau_f, \Upsilon\upsilon_f; 1-i_f, i_f), \\ \upsilon_f = \tilde{W}(\tau_f, \Upsilon\sigma_f; 1-j_f, j_f), \\ \sigma_f = \tilde{W}(\tau_f, \Upsilon\tau_f; 1-k_f, k_f), \end{cases}.$$

It should be noted here that if we choose $k_f = 0,$ in (1.1.3) then we get (1.1.2). Thus, Ishikawa iterative procedure is a special case of Noor iterative procedure.

In the next section, some convergence results in convex GM- spaces are established for some contractive type mapping using Noor iterative procedure. The convergence results of Mann iterations and Ishikawa iteration are also obtained as special cases. The main result generalizes the results of [9, 10, 18].

II. MAIN RESULT

Theorem 2.1: Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \neq \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.1.1)\quad \tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \leq \pi\tilde{\zeta}(\tau,\upsilon,\omega) + \iota\tilde{\zeta}(\tau, \Upsilon\tau, \Upsilon\tau) + o\tilde{\zeta}(\upsilon, \Upsilon\upsilon, \Upsilon\upsilon) + \theta\tilde{\zeta}(\omega, \Upsilon\omega, \Upsilon\omega)$$

$\forall \tau,\upsilon,\omega \in \Im$ and $\pi, o, \theta, \iota \in R^+$ with $0 \leq \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Noor iterative procedure defined by (1.1.3), where $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Noor iteration strongly converges to a fixed point of Υ.

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Proof: Let $\varsigma \in F(\Upsilon)$, then

$$(2.1.2)\quad \tilde{\zeta}(\tau_{f+1},\varsigma,\varsigma)=\tilde{\zeta}\left(\tilde{W}(\tau_f,\Upsilon\upsilon_f;1-i_f,i_f),\varsigma,\varsigma\right) \leq \left(1-i_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+i_f\tilde{\zeta}(\Upsilon\upsilon_f,\varsigma,\varsigma).$$

Now, using (2.1.1), we have

$$\begin{aligned}(2.1.3)\ \tilde{\zeta}(\Upsilon\upsilon_f,\varsigma,\varsigma) &= \tilde{\zeta}(\Upsilon\upsilon_f,\Upsilon\varsigma,\Upsilon\varsigma)\\ &\leq \pi\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\upsilon_f,\Upsilon\upsilon_f,\Upsilon\upsilon_f)\\ &\quad + o\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)+\theta\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)\\ &= \pi\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\upsilon_f,\Upsilon\upsilon_f,\Upsilon\upsilon_f)\\ &\leq \pi\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)\\ &\quad +\iota\left[\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)+\tilde{\zeta}(\varsigma,\Upsilon\upsilon_f,\Upsilon\upsilon_f)\right]\\ &\leq \pi\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)\\ &\quad +\iota\left[\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)+2\tilde{\zeta}(\Upsilon\upsilon_f,\varsigma,\varsigma)\right].\end{aligned}$$

Thus, we get

$$(2.1.4)\quad \tilde{\zeta}(\Upsilon\upsilon_f,\varsigma,\varsigma)\leq\frac{\pi+\iota}{1-2\iota}\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma).$$

Let $\wp=\frac{\pi+\iota}{1-2\iota}$. Then, $0\leq\pi+3\iota<1$ gives $0\leq\wp<1$ with $1-2\iota\neq 0$.

Because if, $1-2\iota=0$, then $0\leq\pi+3\iota<1$ *gives* $\pi<-\iota$, which is a contraction to $\pi\geq 0$.

So, (2.1.4) becomes

$$(2.1.5)\quad \tilde{\zeta}(\Upsilon\upsilon_f,\varsigma,\varsigma)\leq\wp\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma).$$

Thus, combining (2.1.2) and (2.1.5), we get

$$(2.1.6)\quad \tilde{\zeta}(\tau_{f+1},\varsigma,\varsigma)\leq\left(1-i_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+i_f\wp\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma).$$ Now,

$$\begin{aligned}(2.1.7)\quad \tilde{\zeta}(\upsilon_f,\varsigma,\varsigma) &= \tilde{\zeta}\left(\tilde{W}(\tau_f,\Upsilon\sigma_f;1-j_f,j_f),\varsigma,\varsigma\right)\\ &\leq \left(1-j_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+j_f\tilde{\zeta}(\Upsilon\sigma_f,\varsigma,\varsigma).\end{aligned}$$

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Again, using (2.1.1), we have

$$\begin{aligned}
&\tilde{\zeta}(\Upsilon\sigma_f,\varsigma,\varsigma)\\
&=\tilde{\zeta}(\Upsilon\sigma_f,\Upsilon\varsigma,\Upsilon\varsigma)\\
&\le \pi\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\sigma_f,\Upsilon\sigma_f,\Upsilon\sigma_f)\\
&\quad+o\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)+\theta\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)\\
&=\pi\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\sigma_f,\Upsilon\sigma_f,\Upsilon\sigma_f)\\
&\le \pi\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)\\
&\quad+\iota\left[\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)+\tilde{\zeta}(\varsigma,\Upsilon\sigma_f,\Upsilon\sigma_f)\right]\\
&\le p\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)\\
&\quad+\iota\left[\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)+2\tilde{\zeta}(\Upsilon\sigma_f,\varsigma,\varsigma)\right].
\end{aligned}$$

Thus, we get

$$\text{(2.1.8)}\qquad \tilde{\zeta}(\Upsilon\sigma_f,\varsigma,\varsigma)\le\frac{\pi+\iota}{1-2\iota}\tilde{\zeta}(\sigma_f,\varsigma,\varsigma).$$

Using $\wp=\frac{\pi+\iota}{1-2\iota}$, (2.1.8) becomes

$$\text{(2.1.9)}\qquad \tilde{\zeta}(\Upsilon\sigma_f,\varsigma,\varsigma)\le\wp\,\tilde{\zeta}(\sigma_f,\varsigma,\varsigma).$$

Thus, combining (2.1.7) and (2.1.9), we get

$$\begin{aligned}
\text{(2.1.10)}\qquad &\tilde{\zeta}(\upsilon_f,\varsigma,\varsigma)\\
&\le\left(1-j_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+j_f\wp\,\tilde{\zeta}(\sigma_f,\varsigma,\varsigma).
\end{aligned}$$

Using (2.1.6) and (2.1.10), we get

$$\begin{aligned}
\text{(2.1.11)}\ &\tilde{\zeta}(\tau_{f+1},\varsigma,\varsigma)\\
&\le\left(1-i_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&\quad+i_f\wp\left[\left(1-j_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+j_f\wp\,\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)\right]\\
&=\left[1-(1-\wp)i_f-i_fj_f\wp\right]\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&\quad+i_fj_f\wp^2\tilde{\zeta}(\sigma_f,\varsigma,\varsigma).
\end{aligned}$$

Now,

$$\begin{aligned}
\text{(2.1.12)}\qquad &\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)\\
&=\tilde{\zeta}\left(\tilde{W}(\tau_f,\Upsilon\tau_f;1-k_f,k_f),\varsigma,\varsigma\right)\\
&\le\left(1-k_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+k_f\tilde{\zeta}(\Upsilon\tau_f,\varsigma,\varsigma).
\end{aligned}$$

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Again, using (2.1.1), we have

$$\begin{aligned}
&\tilde{\zeta}(\Upsilon\tau_f,\varsigma,\varsigma)\\
&=\tilde{\zeta}(\Upsilon\tau_f,\Upsilon\varsigma,\Upsilon\varsigma)\\
&\le \pi\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\tau_f,\Upsilon\tau_f,\Upsilon\tau_f)\\
&\ +o\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)+\theta\tilde{\zeta}(\varsigma,\Upsilon\varsigma,\Upsilon\varsigma)\\
&=\pi\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+\iota\tilde{\zeta}(\tau_f,\Upsilon\tau_f,\Upsilon\tau_f)
\end{aligned}$$

$$\begin{aligned}
&\le \pi\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&+\iota\left[\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+\tilde{\zeta}(\varsigma,\Upsilon\tau_f,\Upsilon\tau_f)\right]\\
&\le \pi\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&+\iota\left[\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+2\tilde{\zeta}(\Upsilon\tau_f,\varsigma,\varsigma)\right].
\end{aligned}$$

Thus, we get

$$(2.1.13)\qquad \tilde{\zeta}(\Upsilon\tau_f,\varsigma,\varsigma)\le\frac{\pi+\iota}{1-2\iota}\tilde{\zeta}(\tau_f,\varsigma,\varsigma).$$

Using $\wp=\dfrac{\pi+\iota}{1-2\iota}$, (2.1.13) becomes

$$(2.1.14)\qquad \tilde{\zeta}(\Upsilon\tau_f,\varsigma,\varsigma)\le\wp\,\tilde{\zeta}(\tau_f,\varsigma,\varsigma).$$

Thus, combining (2.1.12) and (2.1.14), we get

$$\begin{aligned}
(2.1.15)\qquad &\tilde{\zeta}(\sigma_f,\varsigma,\varsigma)\\
&\le\left(1-k_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)+k_f\wp\,\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&=\left(1-k_f\left(1-\wp\right)\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma).
\end{aligned}$$

Using (2.1.11) and (2.1.15), we get

$$\begin{aligned}
(2.1.16)\ \ &\tilde{\zeta}(\tau_{f+1},\varsigma,\varsigma)\\
&\le\left(1-(1-\wp)i_f-\wp i_f j_f\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&\quad +i_f j_f\wp^2\left(\left(1-k_f+k_f\wp\right)\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\right)\\
&=\left[1-(1-\wp)i_f-\wp(1-\wp)i_f j_f\left(1+k_f\wp\right)\right]\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&\le\left[1-(1-\wp)i_f\right]\tilde{\zeta}(\tau_f,\varsigma,\varsigma)\\
&\le\prod_{h=o}^{f}\left[1-(1-\wp)i_h\right]\tilde{\zeta}(\tau_0,\varsigma,\varsigma)\\
&\le e^{-(1-\wp)\sum_{h=0}^{f}i_h}\,\tilde{\zeta}(\tau_0,\varsigma,\varsigma).
\end{aligned}$$

Since, $\sum_{f=0}^{\infty}i_f=\infty$ and $0\le\wp<1$, this gives $e^{-(1-\wp)\sum_{h=0}^{f}i_h}\to 0.$

Thus, $\lim_{f\to\infty}\tilde{\zeta}(\tau_f,\varsigma,\varsigma)=0.$ Hence, the sequence $\{\tau_f\}$ converges strongly to a fixed point ς of Υ.

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Corollary 2.2: Let $\Upsilon:\Im\to\Im$ be a map with $F(\Upsilon)\neq\phi$ and $(\Im,\tilde{\zeta},\tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.2.1)\ \tilde{\zeta}(\Upsilon\tau,\Upsilon\upsilon,\Upsilon\omega)\leq\pi\tilde{\zeta}(\tau,\upsilon,\omega)+\iota\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau)+o\tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon)+\theta\tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)$$

$\forall\tau,\upsilon,\omega\in\Im$ and $\pi,o,\theta,\iota\in R^+$ with $0\leq\pi+3\iota<1$. Let $\tau_0\in\Im$ and $\{\tau_f\}$ be Ishikawa iterative procedure defined by (1.1.2), where $\sum_{f=0}^{\infty}i_f=\infty$. Then the sequence $\{\tau_f\}$ generated by Ishikawa iteration strongly converges to a fixed point of Υ.

Proof: Choose $k_f=0$ in (1.1.3) to obtain (1.1.2).

Corollary 2.3 [18]Let $\Upsilon:\Im\to\Im$ be a map with $F(\Upsilon)\neq\phi$ and $(\Im,\tilde{\zeta},\tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.3.1)\ \tilde{\zeta}(\Upsilon\tau,\Upsilon\upsilon,\Upsilon\omega)\leq\pi\tilde{\zeta}(\tau,\upsilon,\omega)+\iota\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau)+o\tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon)+\theta\tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)$$

$\forall\tau,\upsilon,\omega\in\Im$ and $\pi,o,\theta,\iota\in R^+$ with $0\leq\pi+3\iota<1$. Let $\tau_0\in\Im$ and $\{\tau_f\}$ be Mann iterative procedure defined by (1.1.1) with $\sum_{f=0}^{\infty}i_f=\infty$. Then the sequence $\{\tau_f\}$ generated by Mann iteration strongly converges to a fixed point of Υ.

Proof: Choose $j_f=0$ and $k_f=0$ in (1.1.3) to obtain (1.1.1).

Corollary 2.3 gives approximation result for the mapping used in Theorem 2.1 of [10].

Corollary 2.4 Let $\Upsilon:\Im\to\Im$ be a map with $F(\Upsilon)\neq\phi$ and $(\Im,\tilde{\zeta},\tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.4.1)\ \tilde{\zeta}(\Upsilon\tau,\Upsilon\upsilon,\Upsilon\omega)\leq\pi\tilde{\zeta}(\tau,\upsilon,\omega)+\iota\left[\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau)+\tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon)+\tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right]$$

$\forall\tau,\upsilon,\omega\in\Im$ and $\pi,\iota\in R^+$ with $0\leq\pi+3\iota<1$. Let $\tau_0\in\Im$ and $\{\tau_f\}$ be Noor iterative procedure defined by (1.1.3) with $\sum_{f=0}^{\infty}i_f=\infty$. Then the sequence $\{\tau_f\}$ generated by Noor iteration strongly converges to a fixed point of Υ.

Proof: Choose $\iota=o=\theta$ in (2.1.1) to obtain (2.4.1).

Corollary 2.5 Let $\Upsilon:\Im\to\Im$ be a map with $F(\Upsilon)\neq\phi$ and $(\Im,\tilde{\zeta},\tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.5.1)\ \tilde{\zeta}(\Upsilon\tau,\Upsilon\upsilon,\Upsilon\omega)\leq\pi\tilde{\zeta}(\tau,\upsilon,\omega)+\iota\left[\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau)+\tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon)+\tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right]$$

$\forall\tau,\upsilon,\omega\in\Im$ and $\pi,\iota\in R^+$ with $0\leq\pi+3\iota<1$. Let $\tau_0\in\Im$ and $\{\tau_f\}$ be Noor iterative procedure defined by (1.1.3) with $\sum_{f=0}^{\infty}i_f=\infty$. Then the sequence $\{\tau_f\}$ generated by Noor iteration strongly converges to a fixed point of Υ.

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Proof: Choose $\iota = o = \theta$ in (2.1.1) to obtain (2.5.1).

Corollary 2.6 Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \neq \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$\begin{aligned}(2.6.1)\ &\tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \\ &\leq \pi\tilde{\zeta}(\tau,\upsilon,\omega) \\ &+ \iota \max\left\{\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right\}\end{aligned}$$

$\forall \tau,\upsilon,\omega \in \Im$ and $\pi, \iota \in R^+$ with $0 \leq \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Noor iterative procedure defined by (1.1.3) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Noor iteration strongly converges to a fixed point of Υ.

Proof: Replace sum by maximum in (2.1.1) to obtain (2.6.1).

Corollary 2.7 Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \neq \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$\begin{aligned}(2.7.1)\ &\tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \\ &\leq \pi\tilde{\zeta}(\tau,\upsilon,\omega) \\ &+ \iota\left[\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right]\end{aligned}$$

$\forall \tau,\upsilon,\omega \in \Im$ and $\pi, \iota \in R^+$ with $0 \leq \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Ishikawa iterative procedure defined by (1.1.2) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Ishikawa iteration strongly converges to a fixed point of Υ .

Proof: Choose $k_f = 0$ in (1.1.3) to obtain (1.1.2) and choose $\iota = o = \theta$ in (2.1.1) to obtain (2.7.1).

Corollary 2.8 Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \neq \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$\begin{aligned}(2.8.1)\ &\tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \\ &\leq \pi\tilde{\zeta}(\tau,\upsilon,\omega) \\ &+ \iota\left[\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right]\end{aligned}$$

$\forall \tau,\upsilon,\omega \in \Im$ and $\pi, \iota \in R^+$ with $0 \leq \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Ishikawa iterative procedure defined by (1.1.2) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Ishikawa iteration strongly converges to a fixed point of Υ.

Proof: Choose $k_f = 0$ in (1.1.3) to obtain (1.1.2) and choose $\iota = o = \theta$ in (2.1.1) to obtain (2.8.1).

Corollary 2.9 Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \neq \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$\begin{aligned}(2.9.1)\ &\tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \\ &\leq \pi\tilde{\zeta}(\tau,\upsilon,\omega) \\ &+ \iota \max\left\{\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right\}\end{aligned}$$

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$\forall \tau, \upsilon, \omega \in \Im$ and $\pi, \iota \in R^{+}$ with $0 \le \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Ishikawa iterative procedure defined by (1.1.2) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Ishikawa iteration strongly converges to a fixed point of Υ.

Proof: Choose $k_f = 0$ in (1.1.3) to obtain (1.1.2). Also, replace sum by maximum in (2.1.1) to obtain (2.9.1).

Corollary 2.10 [18] Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \ne \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.10.1)\ \tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \le \pi\tilde{\zeta}(\tau,\upsilon,\omega) + \iota\left[\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right]$$

$\forall \tau, \upsilon, \omega \in \Im$ and $\pi, \iota \in R^{+}$ with $0 \le \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Mann iterative procedure defined by (1.1.1) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Mann iteration strongly converges to a fixed point of Υ.

Proof: Choose $j_f = 0, k_f = 0$ in (1.1.3) to obtain (1.1.1) and choose $\iota = o = \theta$ in (2.1.1) to obtain (2.10.1).

Corollary 2.10 gives approximation result for the mapping used in Theorem 2.2 of [9].

Corollary 2.11 [18] Let $\Upsilon : \Im \to \Im$ be a map with $F(\Upsilon) \ne \phi$ and $(\Im, \tilde{\zeta}, \tilde{W})$ be a convex GM-space which satisfy the condition

$$(2.11.1)\ \tilde{\zeta}(\Upsilon\tau, \Upsilon\upsilon, \Upsilon\omega) \le \pi\tilde{\zeta}(\tau,\upsilon,\omega) + \iota \max\left\{\tilde{\zeta}(\tau,\Upsilon\tau,\Upsilon\tau) + \tilde{\zeta}(\upsilon,\Upsilon\upsilon,\Upsilon\upsilon) + \tilde{\zeta}(\omega,\Upsilon\omega,\Upsilon\omega)\right\}$$

$\forall \tau, \upsilon, \omega \in \Im$ and $\pi, \iota \in R^{+}$ with $0 \le \pi + 3\iota < 1$. Let $\tau_0 \in \Im$ and $\{\tau_f\}$ be Mann iterative procedure defined by (1.1.1) with $\sum_{f=0}^{\infty} i_f = \infty$. Then the sequence $\{\tau_f\}$ generated by Mann iteration strongly converges to a fixed point of Υ.

Proof: Choose $j_f = 0, k_f = 0$ in (1.1.3) to obtain (1.1.1) and replace sum by maximum in (2.1.1) to obtain (2.11.1).

Corollary 2.11 gives approximation result for the mapping used in Theorem 2.3 of [9].

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