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Using Geometric Methods to Solve the Kinematic Analysis of Complex Mechanical Mechanisms

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***Abstract* - Spatial structures have been used widely for transmission in mechanical engineering, such as a universal jointed shaft in drilling process or a slider-crank linkage in compressing process. Solving kinematic problems in those mechanisms is quite complex. Some solutions have been proposed with both strong and weak points. This study proposed a new method to analyze kinematic problems for mechanisms using orthographic projection. Kinematic analysis of spatial structures could be carried out easily by setting up associative equations as well as simulating the motion of these structures. The simulated results were validated by Matlab software.**

Keywords: Spatial structure; Kinetic analysis; Perpendicular projection; Link equation; Simulation.

I. Introduction

Kinematic analysis is the initial problem to be solved, serving as a basis for subsequent problems such as dynamics, structural strength, machine balancing, optimization, etc.

Various methods exist globally to tackle kinematic analysis problems, including the Denavit-Hartenberg-Craig matrix method, the vector polygon method, and the recursive method. However, these methods tend to be quite complex, especially when applied to spatial mechanisms. For example, the joint separation method yields fewer constraint equations but requires establishing complicated constraint conditions for certain joints. The recursive method provides acceleration expressions that are complex and require suitable computational software.

This paper proposes the use of orthogonal projection to analyze spatial mechanisms. This method is intuitive, straightforward, and avoids cumbersome formulas, simplifying the process of solving kinematic problems.

II. Theoretical Basis for Spatial Mechanism Kinematic Analysis

Each spatial mechanism is a holonomic mechanical system subjected to constraints, expressed as:

$$\sum l_i = 0 \qquad (1)$$

We use generalized coordinates q_1, q_2, q_3, ..., q_f. o define the mechanism's configuration with fff degrees of freedom. Additional redundant generalized coordinates z_1, z_2, z_3,...,z_r are introduced to simplify the establishment of analytical expressions. The constraint equations take the form:

$$f_i\,(q_1,q_2,\ldots,q_f,z_1,z_2,\ldots z_r)=0 \quad (i=1\div r) \qquad (2)$$

$$\text{Where } z_k=z_k(q_1,q_2,\ldots,q_r) \;\; (k=1\div r) \qquad (3)$$

Solving the nonlinear algebraic system (1 & 2) via iterative methods to determine ziz_izi, and their first and second derivatives is central to kinematic analysis.

III. Kinematic Analysis of Four-link rscc Mechanism

The objective is to use orthogonal projection and introduce redundant generalized coordinates to construct the constraint equations. The steps are:

1. Model the mechanical structure into a basic mechanical system.
2. Select a fixed coordinate system OxyzOxyzOxyz and define coordinate systems attached to each link.
3. Project the system and axes onto the xoyxoyxoy plane.
4. Choose the generalized coordinates.
5. Write vector-form constraint equations and project them onto the fixed coordinate system.
6. Determine the direction cosine matrix for each link and establish expressions for position, velocity, and acceleration.

Note:

Choose Oxyz such that: for rotating links, let the Oxyz plane be parallel to the rotation axis; for translating links, align axis x or y with the translation direction.

For each link, identify the angle between the projections of attached coordinate axes and the x, y ,z axes. For prismatic joints, include displacement as a redundant generalized coordinate.

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3.1 Establishing Constraint Equations

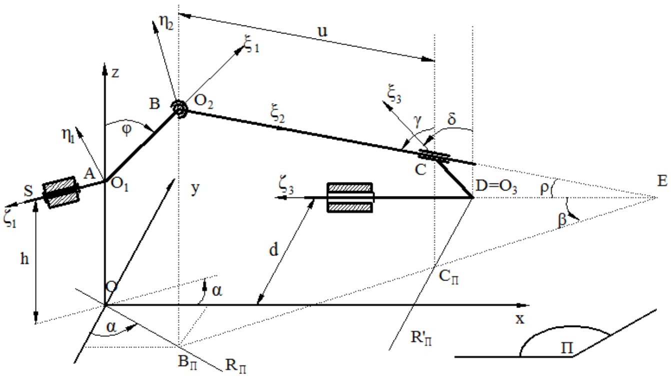

Figure 1: Four-bar RSCC mechanism

Figure 2 illustrates a four-bar RSCC mechanism AB is perpendicular to SA, CD is perpendicular to DE, link SAB rotates around axis SA, link BC moves spatially (rotates and slides) relative to CD, Link CD rotates around and translates along DE. Let $AB=l_1$, $CD=l_2$, DE=a, CE=b.

Define coordinate systems at SAB link: $O_1=A$, direction of ξ_1 as AB, direction of $\zeta_1$as AS, the direction η_1 is chosen so as to form a right-handed coordinate system.

We define a plane Π as follows: Through point D, draw a line parallel to the rotational axis SA of link 1 and another line coinciding with the rotational axis DE. The intersection of these two lines determines the plane Π.

Through O_1, draw a line perpendicular to plane Π that intersects Π at point O; in other words, point O is the orthogonal projection of O_1 onto plane Π.

Choose the coordinate system Oxyz such that: the origin O is the projection of point A onto plane Π, the z-axis is directed along OO_1, the x-axis is aligned with DE, and the y-axis is selected so that Oxyz forms a right-handed coordinate system. As a result, the xoy plane lies on plane Π.

Choose the coordinate system $O_2\xi_2\eta_2\zeta_2$ attached to link BC with O_2 = B, and ξ_2 oriented along BC. The initial position of η_2 can be chosen arbitrarily, provided that it is perpendicular to ξ_2.

Choose the coordinate system $O_3\xi_3\eta_3\zeta_3$ attached to link CDE such that: O_3 is at point D, the ζ_3-axis is directed along ED, the ξ_3-axis is directed along DC, and the η_3-axis is chosen to form a right-handed coordinate system.

Observation: Since link SAB rotates around the fixed axis SA, it always lies in a plane R that passes through point A and is perpendicular to the rotation axis SA. Plane R contains segment OA and is perpendicular to plane Π. According to the properties of orthogonal projection, the projection of plane R onto Π is a straight line R_Π, which forms a constant angle α with the y-axis. The projection of any point on segment AB onto plane Π also lies on this line.

Link CDE rotates around the fixed axis DE, with CD being perpendicular to plane Π. During the rotational motion, CD always lies in a plane R′ that is perpendicular to DE. Since DE lies in plane Π, plane R′ is therefore perpendicular to Π. As a result, the projection of R′ onto Π is a straight line R′_Π. It is clear that R′_Π is perpendicular to DE. The projection of any point located on segment CD onto Π lies on this line R′_Π.

Let the distance between axis x and ζ_3 be denoted as d, and the distance OO_1 as h. In a specific mechanism, these dimensions are completely determined.

According to Gruebler's criterion: $f = \sum_{i=1}^{n_G} f_{Gi} - 6n_L - f_{th}$

With f_{Gi} is the number of degrees of freedom of joint i; n_G is the number of joints; n_L is the number of loops in the chain, f_{th} which is the number of redundant degrees of freedom. For this mechanism, there exists a degree of freedom of movement of link BC rotating around its own axis. This degree of freedom of movement does not affect the kinematic relationship between the input and output of the mechanical system. Therefore, people often call that degree of freedom the redundant degree of freedom.

We have: f=(1+3+3+1)-6-1=1.

Thus, the mechanism has one degree of freedom. We select the independent generalized coordinate as φ, and the dependent generalized coordinates as β(φ), γ(φ), δ(φ), u(φ), and xD(φ).

Due to the chosen coordinate system, the angle α is constant and C'D is always perpendicular to the x-axis; therefore, two generalized coordinates can be eliminated if the xoy plane is chosen arbitrarily.

From the diagram, we derive the constraint equation of the mechanism in vector form:

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DO} = 0 \qquad (4)$$

$$\overrightarrow{DC} + \overrightarrow{CE} + \overrightarrow{ED} = 0 \qquad (5)$$

Projecting onto the xyz coordinate axes, we obtain the following equations:

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$$\begin{aligned} f_1 &= l_1 \sin\varphi \sin\alpha + u\cos\beta\sin\gamma - x_D = 0 \\ f_2 &= -l_1 \sin\varphi\cos\alpha + u\sin\beta\sin\gamma + l_2\sin\delta - d = 0 \\ f_3 &= h + l_1\cos\varphi - u\cos\gamma \text{-} l_2\cos\delta = 0 \\ f_4 &= b\cos\rho - a = 0 \\ f_5 &= \text{-}l_2\sin\delta + b\sin\beta\sin\gamma = 0 \\ f_6 &= l_2\cos\delta - b\cos\gamma = 0 \end{aligned} \tag{6}$$

We have an equation involving constants that always holds true for the right triangle CDE.

Therefore, the system of constraint equations for the mechanism can be written as follows:

$$\begin{aligned} f_1 &= l_1 \sin\varphi \sin\alpha + u\cos\beta\sin\gamma - x_D = 0 \\ f_2 &= -l_1 \sin\varphi\cos\alpha + u\sin\beta\sin\gamma + l_2\sin\delta - d = 0 \\ f_3 &= h + l_1\cos\varphi - u\cos\gamma \text{-} l_2\cos\delta = 0 \\ f_4 &= \text{-}l_2\sin\delta + b\sin\beta\sin\gamma = 0 \\ f_5 &= l_2\cos\delta - b\cos\gamma = 0 \end{aligned} \tag{7}$$

3.2 Solving the Inverse Kinematics Problem

$$\begin{aligned} \dot f_1 &= l_1\dot\varphi\cos\varphi\sin\alpha - u\dot\beta\sin\beta\sin\gamma + u\dot\gamma\cos\beta\cos\gamma - 1 = 0 \\ \dot f_2 &= -l_1\dot\varphi\cos\varphi\cos\alpha + u\dot\beta\cos\beta\sin\gamma + u\dot\gamma\sin\beta\cos\gamma + l_2\dot\delta\cos\delta - d = 0 \\ \dot f_3 &= h - l_1\dot\varphi\sin\varphi + u\dot\gamma\sin\gamma + l_2\dot\delta\sin\delta = 0 \\ \dot f_4 &= -l_2\dot\delta\cos\delta + b\dot\beta\cos\beta\sin\gamma + b\dot\gamma\sin\beta\cos\gamma \\ \dot f_5 &= b\dot\delta\sin\delta - l_2\dot\delta\sin\delta \end{aligned} \tag{8}$$

From equation (8), we derive the Jacobian matrices:

$$\mathbf{J}_x = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{J}_q = \begin{bmatrix} l_1\cos\varphi\sin\alpha & -u\sin\beta\sin\gamma & u\cos\beta\cos\gamma & 0 & \cos\beta\sin\gamma \\ -l_1\cos\varphi\cos\alpha & u\cos\beta\sin\gamma & u\sin\beta\cos\gamma & l_2\cos\delta & \sin\beta\sin\gamma \\ -l_1\sin\varphi & 0 & u\sin\gamma & l_2\sin\delta & -\cos\gamma \\ 0 & \text{b}\cos\beta\sin\gamma & \text{b}\sin\beta\cos\gamma & -l_2\cos\delta & 0 \\ 0 & 0 & \text{b}\sin\gamma & -l_2\cos\delta & 0 \end{bmatrix} \tag{9}$$

According to equation (1.4), the formula for determining the joint velocities is:

$$\dot{\mathbf{q}} = -\mathbf{J}_q^{-1}\mathbf{J}_x\dot{\mathbf{x}} \tag{10}$$

According to equation (1.6), the time derivatives of the Jacobian matrices can be calculated as follows:

$$\dot{\mathbf{J}}_x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

$$\dot{\mathbf{J}}_q = \begin{bmatrix} -l_1\dot\varphi\sin\varphi\sin\alpha & -u\dot\beta\cos\beta\sin\gamma - u\dot\gamma\sin\beta\cos\gamma - \dot u\sin\beta\sin\gamma & \dots \\ l_1\dot\varphi\sin\varphi\cos\alpha & -u\dot\beta\sin\beta\sin\gamma + u\dot\gamma\cos\beta\cos\gamma + \dot u\cos\beta\sin\gamma & \dots \\ -l_1\dot\varphi\cos\varphi & 0 & \dots \\ 0 & -b\dot\beta\sin\beta\sin\gamma + b\dot\gamma\cos\beta\cos\gamma & \dots \\ 0 & 0 & \dots \end{bmatrix}$$

$$\begin{bmatrix} -u\dot\beta\sin\beta\cos\gamma - u\dot\gamma\cos\beta\sin\gamma + \dot u\cos\beta\cos\gamma & 0 & -\dot\beta\sin\beta\sin\gamma + \dot\gamma\cos\beta\cos\gamma \\ u\dot\beta\cos\beta\cos\gamma - u\dot\gamma\sin\beta\sin\gamma + \dot u\sin\beta\cos\gamma & -l_2\dot\delta\sin\delta & \dot\beta\cos\beta\sin\gamma + \dot\gamma\sin\beta\cos\gamma \\ u\dot\gamma\cos\gamma + \dot u\sin\gamma & l_2\dot\delta\cos\delta & \dot\gamma\sin\gamma \\ b\dot\beta\cos\beta\cos\gamma - b\dot\gamma\sin\beta\sin\gamma & l_2\dot\delta\sin\delta & 0 \\ b\dot\gamma\cos\gamma & -l_2\dot\delta\cos\delta & 0 \end{bmatrix}$$

(12)

According to equation (1.8), the formula for determining the joint accelerations is:

$$\ddot{\mathbf{q}} = -\mathbf{J}_q^{-1}(\mathbf{J}_x\ddot{\mathbf{x}} + \dot{\mathbf{J}}_x\dot{\mathbf{x}} - \dot{\mathbf{J}}_q\mathbf{J}_q^{-1}\mathbf{J}_x\dot{\mathbf{x}}) \tag{13}$$

By applying the Newton–Raphson method described above, we sequentially compute the positions, velocities, and accelerations of the joints using the following parameter values:

$$h = 10cm, l_1 = 10cm, l_2 = 11cm, b = 15cm, \text{d} = 10\,\text{cm}, \text{e} = 15\,\text{cm}, \alpha = 75^0(1.3090)$$

The selected input parameter set is as follows:

$$\varphi_0 = 30^0(0.5236\,\text{rad}), \beta_0 = 0.5016(\text{rad}), \gamma_0 = 0.8995(\text{rad}),$$
$$\delta_0 = 0.5443(\text{rad}), \text{u} = 15\,\text{cm}, \text{x}_D = 15cm$$

Velocity of the driving link:

$$\dot\varphi = 2\,pi(\text{rad/ s})$$

Using Matlab software, the inverse kinematics problem of the four-bar RSCC mechanism can be easily solved, and some of the results are presented in the graphs from Figure 2 to Figure 11.

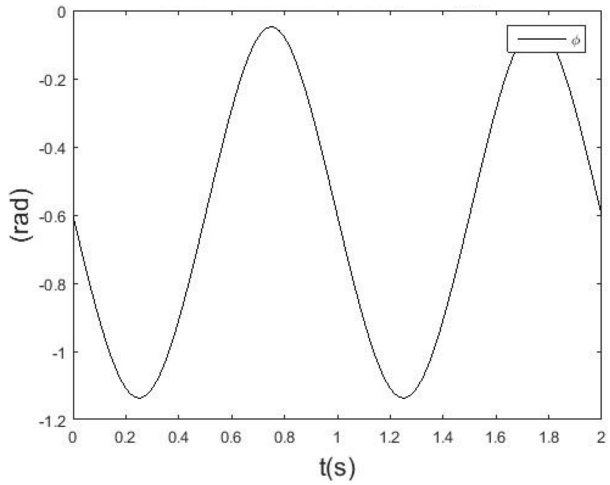

Figure 2: Graph of coordinate φ

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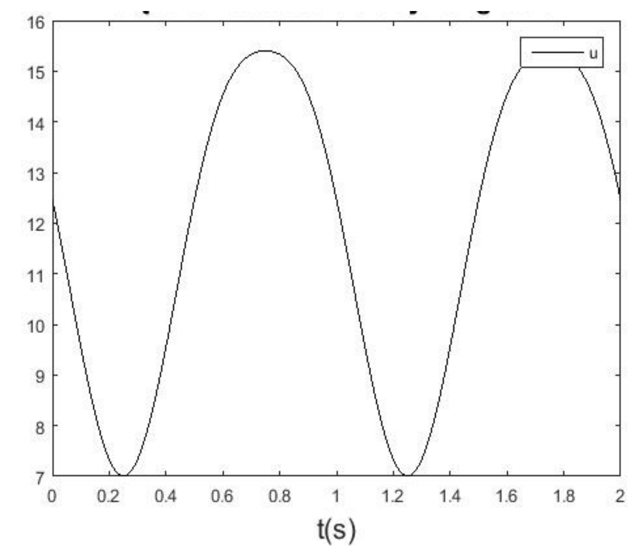

Figure 3: Graph of coordinate u

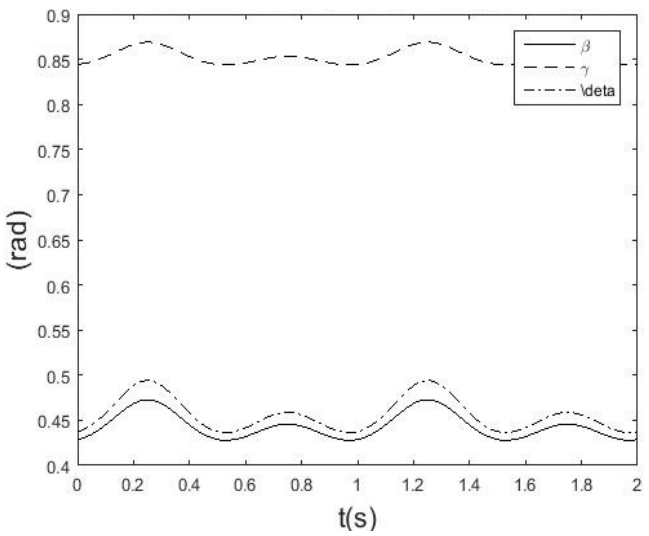

Figure 4: Graph of coordinate βγ δ

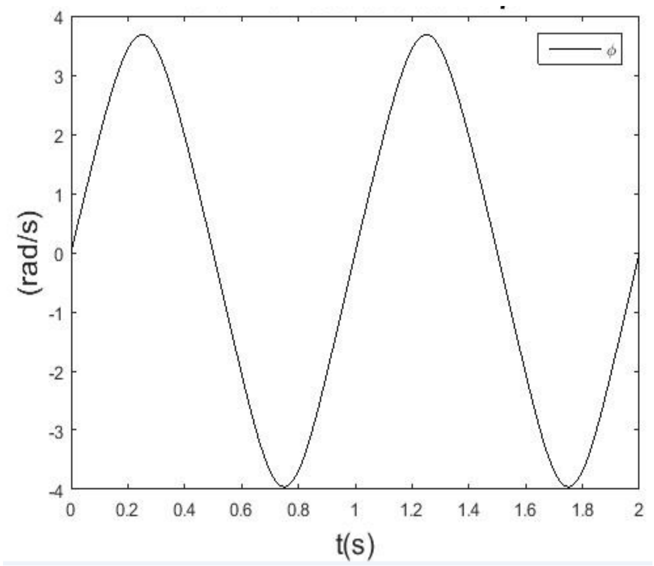

Figure 5: Graph of velocity φ

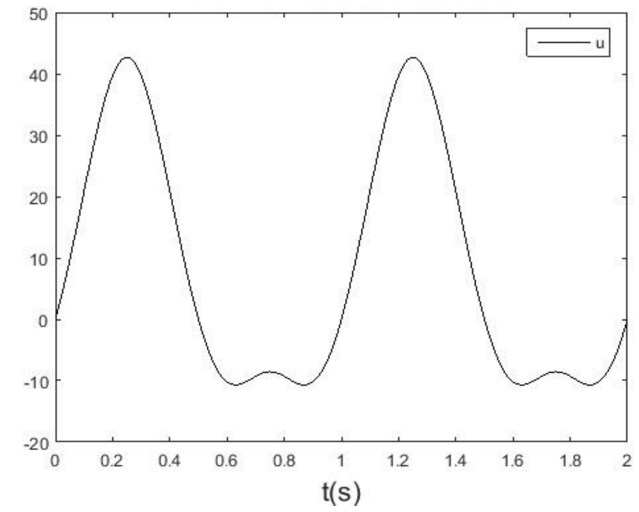

Figure 6: Graph of velocities u

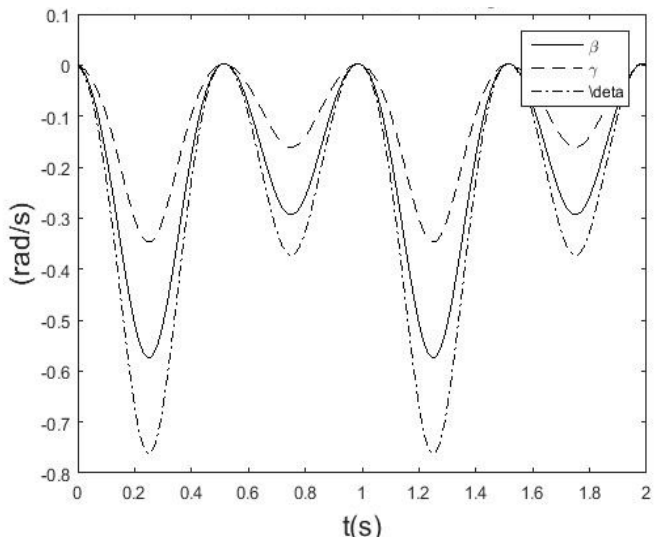

Figure 7: Graph of velocities βγ δ

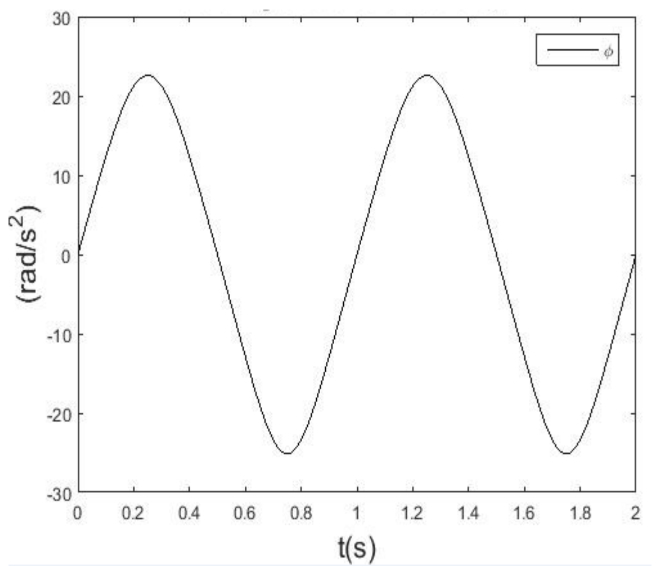

Figure 8: Graph of acceleration φ

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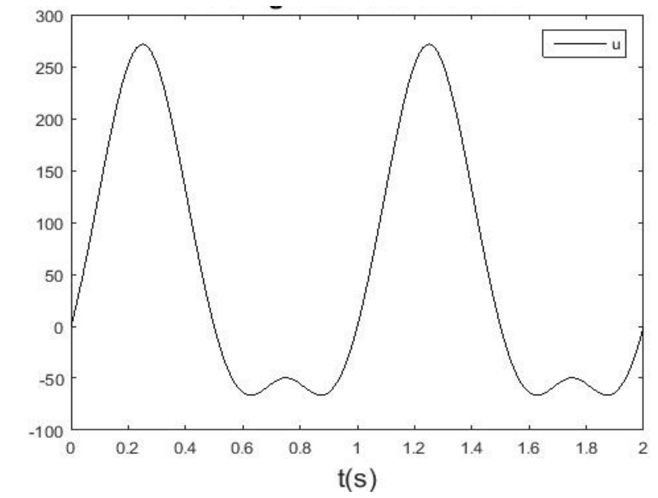

Figure 9: Graph of acceleration u

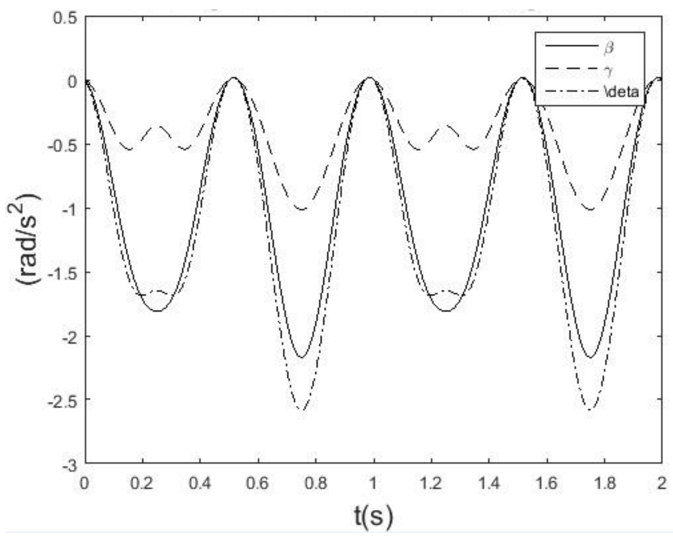

Figure 10: Graph of acceleration βγ δ

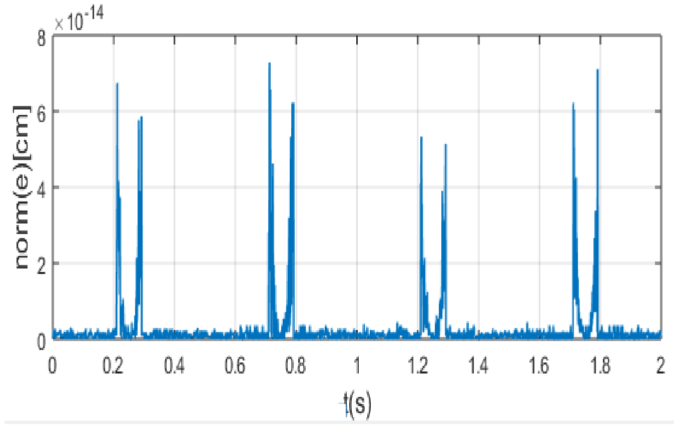

Figure 11: Constraint equation error

IV. Conclusions

This paper presents the orthographic projection method to solve the problem of kinematic analysis of spatial mechanical systems. This method is simple and can be easily applied in practical manufacturing. In addition to outlining the steps for conducting kinematic analysis of spatial mechanisms using the orthographic projection method, the paper also applies the method to analyze the kinematics of a spatial mechanism consisting of two revolute joints, one Cardan joint, and one spherical joint.

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