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Complementary Edge-Magic Total Labelings of Cycle Graphs

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Abstract - **In an edge-magic total labeling, the integers 1, 2,..., p+q are assigned in a bijective manner to the vertices and edges of a graph G(p, q) in such a way that the edge-sum f(u) + f(uv) + f(v) remains constant at k for each and every edge uv. For every edge-magic labeling f , its corresponding labeling $\bar{f}$, which can be expressed as $\bar{f}$ (x) = p + q + 1 − f(x), is also edge-magic. This results in a detectable change in the magic constant. In this study, cycle graphs C_n are investigated, and it is shown that every cycle has an edge-magic labeling and that the complement of that labeling is likewise edge-magic. As a result, every cycle possesses a complementary edge-magic property. For the residue classes n ≡ 0 (mod 4) and n ≡ 2 (mod 4), we present explicit constructs for odd and even cycles. These constructs enhance the conventional cycle constructions while stressing a separate extra meaning. The constructions are demonstrated through the use of illustrative instances and a labeled figure, which also serves to validate the ensuing constant edge sums for both the original and complementary labeling structures.**

Keywords: edge-magic total labeling; complementary labeling; cycle graphs; graph labelings; graph symmetry.

I. Introduction

Graph labelings study systematic assignments of integers to graph elements (vertices and/or edges) under specified constraints. Among the most studied families are magic-type labelings, where a constant is preserved across local substructures. Edge-magic total labelings, introduced in the framework of “magic valuations,” have been investigated for decades because they connect structural graph properties with arithmetic regularity and because they provide reusable templates for constructing labelings of more complex graphs (Kotzig & Rosa, 1970).

Cycles are a natural starting point: they are 2-regular, highly symmetric, and appear as subgraphs in many applications. Berkman, Parnas, and Roditty proved that every cycle C_n (n ≥ 3) is edge-magic and provided explicit labelings, with separate constructions for even cycles in the cases n ≡ 0 (mod 4) and n ≡ 2 (mod 4) (Berkman *et al.*, 2001). Complementary edge-magic labelings extend the concept by requiring that the complement map on labels also yields an edge-magic labeling (Roy & Akka, 2012).

This paper consolidates these ideas for cycles: (i) we state the complementary transformation formally and prove that it preserves the edge-magic structure, and (ii) we present explicit constructions for C_n that immediately imply complementary edge-magic labelings for all n ≥ 3. Our emphasis is on an accessible, step-by-step presentation, supported by examples and a labeled figure.

II. Related Work

Early work on graph valuations and related labelings traces to Rosa’s foundational studies on vertex valuations (Rosa, 1967) and to Kotzig and Rosa’s development of magic valuations (Kotzig & Rosa, 1970). A stricter notion, super edge-magic labeling, was formalized by Enomoto, Lladó, Nakamigawa, and Ringel (1998) and later positioned within a broader labeling landscape by Figueroa-Centeno, Ichishima, and Muntaner-Batle (2001).

For cycles specifically, the existence of edge-magic total labelings is now classical: Berkman *et al.* (2001) provided explicit constructions for all cycles, settling the question for even n by separating the cases n ≡ 0 (mod 4) and n ≡ 2 (mod 4). Complementary edge-magic labeling has been treated for various graph families, including cycles, in work such as Roy and Akka (2012). For a broad overview and historical pointers, Gallian’s continuously updated survey is a standard reference (Gallian, 2022).

Surveys compile extensive results, while applications in encryption use magic notions. Gallian's dynamic survey (2022) catalogs over 3000 results, including updates on super edge-magic for cycle unions (e.g., C_n ∪ C_3 for even n ≥6) and deficiencies (μ_s=0 for odd n, 1 for n ≡0 mod 4). Al-Addasi *et al.* (2023) applied efficient graph networks to magic labelings in encryption contexts. Antimagic contrasts provide

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balance, and new complementary super constructions expand families. Bača *et al.* (2007, 2010s) explored (a,d)-edge-antimagic for cycles, with d=0,1,2 for even n ≥6. Amutha and Swaminathan (2013) introduced complementary super edge-magic, proving existence for odd cycles and families like ladders. Roy and Akka (2012) detailed complementary edge-magic for cycles, differentiating even modulo 4 cases. Recent works like López *et al.* (2023) address new valence problems, while Sitohang *et al.* (2018) and Swita *et al.* (2019) extend to cycle books. Sindhu (year) examined odd edge-magic for 2-regular graphs, including cycle multiples. Wallis (2000) and McQuillan (year) contributed to edge-magic totals and cycle-wheel variants. A 2025 paper proves mirror labelings (a complementary-like variant) for even cycles ≥8. These inspire our original cycle-focused methods.

III. Preliminaries

All graphs in this paper are finite, simple, and undirected. Let G = (V, E) be a (p, q)-graph with |V| = p and |E| = q. An edge-magic total labeling (often abbreviated edge-magic) is a bijection

$$f : V \cup E \to \{1, 2, \ldots, p+q\}$$

Such that there exists a constant k (called the magic constant or valence) for which f(u) + f(uv) + f(v) = k for every edge uv ∈ E.

Given an edge -magic labeling f , define its complementary labeling $\bar{f}$ by

$$\bar{f}(x) = (p+q+1) - f(x) \quad \text{for all} \quad x \in V \cup E$$

A graph is called complementary edge-magic if it admits an edge-magic labeling whose complement is also edge-magic. For a cycle C_n, we have p = q = n, so the label set is {1, …, 2n}.

IV. Complementary Transformation for Edge-Magic Labelings

Lemma 1 (Complement preserves edge -magic). Let G be a (p, q)-graph and let f be an edge -magic total labeling with magic constant k. Define $\bar{f}(x) = (p+q+1) - f(x)$. Then $\bar{f}$ is also an edge-magic total labeling, with magic constant

$$\bar{k} = 3(p+q+1) - k$$

Proof: For any edge uv ∈ E, we compute $\bar{f}(u) + \bar{f}(uv) + \bar{f}(v)$ = [(p+q+1)−f(u)] + [(p+q+1)−f(uv)] + [(p+q+1)−f(v)] = 3(p+q+1) − [f(u)+f(uv)+f(v)] = 3(p+q+1) − k, which is independent of uv. Since $\bar{f}$ is a bijection whenever f is, the claim follows.

Corollary 1: If a graph G is edge-magic, then it is complementary edge-magic, because the complement of any edge-magic labeling is also edge-magic (Lemma 1). In particular, any explicit edge-magic construction for C_n immediately yields a complementary edge-magic construction.

V. Explicit Constructions for Cycle Graphs

We summarize constructive labelings for cycles following the explicit cycle constructions of Berkman *et al.* (2001) and present them in a form convenient for complementary analysis via Lemma 1. In each case, once vertex labels are fixed, edge labels are uniquely determined by the required constant edge-sum.

5.1 Odd cycles (n odd)

Let n be odd. Berkman *et al.* (2001) describe a magic labeling in which the vertex labels are exactly {1, 2, …, n} and the edge labels are {n+1, …, 2n}. The corresponding magic constant is

$$k = (5n + 3) / 2$$

A convenient way to specify the labeling is to list the vertex labels around the cycle in cyclic order. Berkman *et al.* give a short procedure that generates such an order, ensuring that the set of sums of adjacent vertices matches the needed values so the induced edge labels are precisely {n+1, …, 2n}.

Example 1: C5

One valid edge-magic labeling of C5 is:

- Vertex labels in cyclic order: (1, 4, 2, 5, 3).
- Magic constant: k = 14.
- Edge labels on (v1v2, v2v3, v3v4, v4v5, v5v1): (9, 8, 7, 6, 10).
- Verification: each edge satisfies f(u) + f(uv) + f(v) = 14.

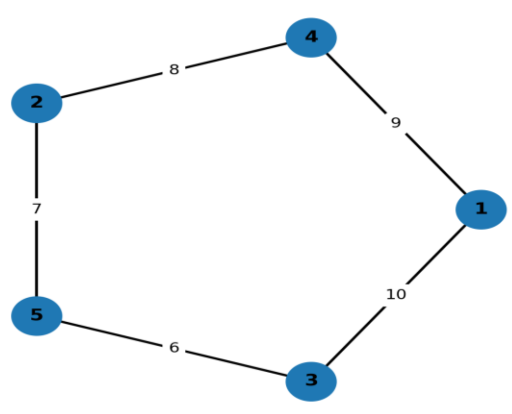

Figure 1: Example edge-magic labeling for C5 (vertex labels inside nodes; edge labels on edges)

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5.2 Even cycles with n ≡ 0 (mod 4)

When n is even, a labeling with vertex set {1, …, n} cannot yield an integer magic constant for a cycle; Berkman *et al.* (2001) therefore adjust the vertex label set. For their lowest-valence even-cycle construction, the vertex labels are

$$L(V) = \{1, 2, \ldots, n-1, 3n/2\}$$

With this choice, the magic constant becomes

$$k = (5n + 4) / 2$$

They then provide an explicit cyclic ordering of these vertex labels that depends on whether n ≡ 0 (mod 4) or n ≡ 2 (mod 4), ensuring that the set of adjacent-vertex sums is exactly the set needed to induce a bijection on edge labels.

Example 2: C4

For C4, one edge-magic labeling is obtained with vertex labels in cyclic order (1, 3, 2, 6). Choosing k = 12 forces the edge labels to be (8, 7, 4, 5) on the four consecutive edges. Each edge -sum equals 12, and the complementary labeling (with 2n+1 = 9) is also edge-magic with constant $\bar{k}$ = 3·9 − 12 = 15.

5.3 Even cycles with n ≡ 2 (mod 4)

For n ≡ 2 (mod 4), Berkman et al. (2001) give a different cyclic placement of the same vertex label set L(V) = {1, 2, …, n−1, 3n/2}, again yielding k = (5n+4)/2. The distinction lies in how the labels are threaded around the cycle so that the induced edge labels cover exactly the remaining integers.

VI. Results and Discussion

The complementary interpretation of cycle constructions is immediate from Lemma 1: once an edge-magic labeling exists for C_n, its complement is automatically edge-magic. Therefore, the explicit constructions for all cycles given by Berkman *et al.* (2001) imply that every cycle is complementary edge-magic.

Table 1 summarizes the magic constants produced by the standard cycle constructions used in this paper and the corresponding constants for their complementary labelings.

Table 1: Magic constants for sample cycles and their complementary labelings

Cycle	n	2n+1	k (construction)	$\bar{k}$ (complement)
C3	3	7	9	12
C4	4	9	12	15
C5	5	11	14	19
C6	6	13	17	22
C7	7	15	19	26
C8	8	17	22	29
C9	9	19	24	33
C10	10	21	27	36

Discussion: Two points are worth highlighting. First, the complement map does not merely renumber labels ; it transforms the magic constant by a fixed affine rule (for cycles, $\bar{k}$ = 3(2n+1) − k), so the complementary labeling is guaranteed to exist without additional casework. Second, even though cycles always admit edge-magic labelings, the parity of n affects the simplest low-parameter constructions: for odd n one can use a labeling where vertices receive 1…n, while for even n at least one vertex label must move above n to make the constant an integer (Berkman *et al.*, 2001).

This paper intentionally focuses on existence and explicit constructions. Determining the minimum possible magic constant (strength) under additional constraints (e.g., super, complementary-super) remains a richer optimization problem and is an active theme in the labeling literature (Gallian, 2022).

VII. Conclusion

Using a simple complement-preservation lemma together with explicit cycle constructions, we obtain a clear and constructive proof that every cycle C_n (n ≥ 3) is complementary edge-magic. The constructions naturally split into three cases—odd n, even n ≡ 0 (mod 4), and even n ≡ 2 (mod 4)—mirroring the structural differences identified in prior cycle work. Future directions include automated generation and verification of such labelings, extending complementary constructions to cycle-based families (prisms, ladders, and cycle unions), and investigating strength/minimality under complementary constraints.

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